On the Domain of Nörlund Matrix in the Space bv of Bounded Variation Sequences

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Abstract: In this paper, we introduce a new sequence space $bv(N^t)$ as the domain of Nörlund matrix N^t in the space of all sequences of bounded variation. Firstly, we give some topological properties and inclusion relations. Moreover, we determine the $\alpha-, \beta-$ and $\gamma-$duals of the space $bv(N^t)$. Finally, we characterize some new matrix classes over the space $bv(N^t)$ into some classical sequence space and vice versa.

Keywords: Bounded Variation, Nörlund Mean, Sequence Spaces, Matrix Domain, Matrix Transformation

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1. Introduction

We denote the space of all real valued sequences by ω. Each vector subspace of ω is called as a sequence space as well. The spaces of all bounded, convergent and null sequences are denoted by ℓ_∞, c and c_0, respectively. By ℓ_1, ℓ_p, cs, cs_0 and bs, we denote the spaces of all absolutely convergent, p-absolutely convergent, convergent, convergent to zero and bounded series, respectively; where $1 < p < \infty$.

A linear topological space λ is called a K-space if each of the map $p_i : \lambda \longrightarrow \mathbb{C}$ defined by $p_i(x) = x_i$ is continuous for all $i \in \mathbb{N}$, where $\mathbb{C}$ denotes the complex field and $\mathbb{N} = \{0,1,2,3,\dots\}$. A K-space λ is called an FK-space if λ is a complete linear metric space. If an FK-space has a normable topology then it is called a *BK*-space. If λ is an *FK*-space, $\Phi \subset \lambda$ and (e^k) is a basis for λ then λ is said to have *AK* property, where (e^k) is the sequence whose only non-zero term is a 1 in k^{th} place for each $k \in \mathbb{N}$ and $\Phi = span\{e^k\}$. If Φ is dense in λ, then λ is called *AD*-space, thus *AK* implies *AD*.

Let λ and μ be two sequence spaces, and $A = (a_{nk})$ be an infinite matrix of real or complex numbers, where $n, k \in \mathbb{N}$. For every sequence $X = (x_k) \in \lambda$ the sequence $Ax = Ax = ((Ax)_n) \in \mu$ is called A-transform of x, where

(1.0) $$(Ax)_{\mathrm{n}} = \sum_{k=0}^{\infty} a_{nk}\, x_k \,.$$

Then, A defines a matrix mapping from $\lambda\ to\ \mu$ and we show it by writing $A{:}\,\lambda \longrightarrow \mu$.

By $A \in (\lambda : \mu)$, we denote the class of all matrices A such that $A : \lambda \longrightarrow \mu$ if and only if the series on the right side of (1.0) converges for each $\mathrm{n} \in \mathbb{N}$ and every $\mathrm{x} \in \lambda$, and we have $Ax = ((Ax)_n)$ belongs to μ for all $\mathrm{x} \in \lambda$. A sequence x is said to be A −summable to l and is called as the A −limit of x.

Let λ be a sequence space and A be an infinite matrix. The matrix domain λ_A of A in λ is defined by

$$\lambda_A = \{x = (x_k) \in \omega : \ Ax \in \lambda\}$$

which is a sequence space.

Let (t_k) be a nonnegative real sequence with $\mathrm{t}_0 > 0$ and $T_n = \sum_{k=0}^{n} t_k$ for all $\in \mathbb{N}$. Then, the Nörlund mean with respect to the sequence $t = (t_k)$ is defined by the matrix $N^t = (a_{nk}^t)$ as follows

(1.1) $$a_{nk}^t = \begin{cases} \frac{t_{n-k}}{T_n} & , \ 0 \le k \le n \\ 0 & , \ k > n \end{cases}$$

For every , $\mathrm{n} \in \mathbb{N}$. It is know that the Nörlund matrix N^{t} is a Teoplitz matrix if and only if $\frac{t_n}{T_n} \longrightarrow 0, as\ n \longrightarrow \infty$. Furthermore, if we take $t = e = (1,1,1,\dots)$, then the Nörlund matrix N^{t} is reduced to Cesàro mean C_1 of order one and if we choose $t_n = A_n^{r-1}$ for every $\mathrm{n} \in \mathbb{N}$, then the N^{t} Nörlund mean becomes Cesàro mean C_r of order r, where $\mathrm{r} > -1$ and

$$A_n^t = \begin{cases} \dfrac{(r+1)(r+2)\dots(r+n)}{n!} & , \quad n = 1,2,3,\dots \\ 0 & , \quad n = 0 \end{cases}$$

Let $\mathrm{t}_0 = \mathrm{D}_0 = 1$ and define D_n for $\mathrm{n} \in \{1,2,3,\dots\}$ by

(1.2) $$D_n = \begin{vmatrix} t_1 & 1 & 0 & 0 & \cdots & 0 \\ t_2 & t_1 & 1 & 0 & \cdots & 0 \\ t_3 & t_2 & t_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ t_{n-1} & t_{n-2} & t_{n-3} & t_{n-4} & \cdots & 1 \\ t_n & t_{n-1} & t_{n-2} & t_{n-3} & \dots & t_1 \end{vmatrix}$$

with $D_1 = t_1\,, D_2 = (t_1)^2 - t_2\,, D_3 = (t_3)^3 - 2t_1t_2 + t_3 \dots \dots$ which is generalized by

$$D_k = \sum_{j=1}^{k-1} (-1)^{j-1} t_j D_{k-j} + (-1)^{k-1} t_k$$

then the inverse matrix $U^t = (u_{nk}^t)$ of Nörlund matrix N^{t} was defined by Mears, F. M. (1943) for all k, $\mathrm{n} \in \mathbb{N}$ as follows

(1.3) $$u_{nk}^t = \begin{cases} (-1)^{n-k} D_{n-k} T_k & , \ 0 \le k \le n, \\ 0 & , \ k > n. \end{cases}$$

The sequence space of all sequences of bounded variation defined as in the following set

$$bv = \left\{ x = (x_k) \in \omega : \sum_{k=1}^{\infty} |x_k - x_{k-1}| < \infty \right\}$$

which is BK −space under the norm $\|x\|_{bv} = |x_0| + \sum_{k=1}^{\infty}|x_k - x_{k-1}|$ for $x \in bv$. It is well-known that the inclusion $l_1 \subset bv \subset c$ hold.

The α, β and γ-duals of the sequence space λ, repectively as

$$\lambda^{\alpha} = D(\lambda, \ell_1) = \{a = (a_k) : \ ax = (a_k x_k) \in \ell_1 \ for \ all \ x \in \lambda\}$$

$$\lambda^{\beta} = D(\lambda, cs) = \{a = (a_k) : \ ax = (a_k x_k) \in cs \ \ for \ all \ x \in \lambda\}$$

$$\lambda^{\gamma} = D(\lambda, bs) = \{a = (a_k) : \ ax = (a_k x_k) \in bs \ \ for \ all \ x \in \lambda\}$$

The α, β and γ-duals of the space bv is defined by

$$bv^{\alpha} = l_1, \quad bv^{\beta} = cs, \quad bv^{\gamma} = bs$$

2. The Sequence Space $bv(N^t)$

We introduce the sequence space $bv(N^t)$ as the set of all sequences of N^t − transform of all sequences of bounded variation as;

$$bv(N^t) \ = \left\{ x = (x_k) \in \omega : \sum_{k=1}^{\infty} \left| \frac{1}{T_k} \sum_{j=0}^{k} t_{k-j}\, x_j - \frac{1}{T_{k-1}} \sum_{j=0}^{k-1} t_{k-j-1}\, x_j \right| < \infty \right\}$$

N^t − transform of $x = (x_k)$ defined by

$$y_k = \sum_{j=0}^{k-1} \left(\frac{1}{T_k} t_{k-j} x_j - \frac{1}{T_{k-1}} t_{k-j-1} x_j \right) + \frac{t_0}{T_k} x_k$$

$$y_k = \sum_{j=0}^{k-1} \left(\frac{1}{T_k} t_{k-j} - \frac{1}{T_{k-1}} t_{k-j-1} \right) x_j + \frac{1}{T_k} x_k$$

(2.1) $$x_k = \textstyle\sum_{j=0}^{k-1} (-1)^{k-j-1} D_{k-j-1} T_{k-1} y_j + T_k y_k$$

for all $x = (x_k) \in bv(N^t)$. The relation between $x = (x_k)$ and $y = (y_k)$ is shown by (2.1). For the sake of brevity in notation, we may also write here and after that

(2.2) $$y_k = \sum_{j=0}^{k-1} \left(\Delta \left(\frac{t_{k-j}}{T_k} \right) \right) x_j + \frac{1}{T_k} x_k$$

Theorem 2.1. The sequence space $bv(N^t)$ is *BK*-space with the norm given by

$$\|x\|_{bv(N^t)} = \|N^t x\|_{bv} = |x_0| + \sum_{k=1}^{\infty} \left| \frac{1}{T_k} \sum_{j=0}^{k} t_{k-j}\, x_j - \frac{1}{T_{k-1}} \sum_{j=0}^{k-1} t_{k-j-1}\, x_j \right|$$

Proof. Since the space bv is a *BK*-space with the norm $\|x\|_{bv} = |x_0| + \sum_{k=1}^{\infty}|x_k - x_{k-1}|$ for $x \in bv$ and the Nörlun matrix is triangular matrix, we can easily say that the sequence space $bv(N^t)$ is a BK-space with the norm $\|x\|_{bv(N^t)} = \|N^t x\|_{bv}$.

Corollary 2.2. The sequence space $bv(N^t)$ is linearly norm isomorphic to the space bv.

It is well-known from Theorem 2.3 of Jarrah and Malkowsky (1998) that the infinite matrix domain λ_A of an infinite matrix $A = (a_{nk})$ in a normed sequence space λ has a basis if and only if λ has a basis, if *A* is a triangular matrix. As a direct consequence of this fact, we have:

Corollary 2.3. Let define the sequence $q^{(k)} = \left\{q_n^{(k)}\right\}_{n\in\mathbb{N}}$ of alements of the sequence space $bv(N^t)$ for every fixed $k \in \mathbb{N}$ by

$$q_n^{(k)} = \begin{cases} (-1)^{n-k-1} D_{n-k-1} T_{n-1} \,, & 1 \le k < n \\ T_n \,, & k = n \\ 0 \,, & k > n \end{cases}$$

Therefore, the sequence $\{q^{(k)}\}_{k\in\mathbb{N}}$ is a basis for the space $bv(N^t)$ and for any $x \in bv(N^t)$ has a unique representation of the form

$$x = \sum_k y_k\, q^{(k)}$$

where $y = (y_k)$ is $\boldsymbol{N^t} -$ transform of $x = (x_k)$ defined by the equality (2.1).

Lemma 2.4. [3] The matrix $A \in (bv: bv)$ if and only if

$$\sup_k \sum_{n=0}^{\infty} \left| \sum_{j=k}^{\infty} (a_{nj} - a_{n-1,j}) \right| < \infty$$

Theorem 2.5. The inclusion $bv \subset bv(N^t)$ strictly hold.

Proof. Since the Nörlund matrix is a triangular and it satisfy the condition of the Lemma 2.4. Then we may say that $(N^t x)_k \in bv$ for all $x = (x_k) \in bv$ which shows that the inclusion $bv \subset bv(N^t)$ hold. Now, in order to show that the inclusion is strict, we should define a sequence which is in the sequence space of N^t –bounded variation but not in the sequence space of bounded variation. Let define a sequence $x = (x_k)$ as

$$x_k = \sum_{j=0}^{k} (-1)^{k-j} D_{k-j} T_j$$

for all $k \in \mathbb{N}$. Then it is abvious that the sequence $x = (x_k)$ is not in the space bv. But the partial sum of $(N^t x)_k = 1$ for all $k \in \mathbb{N}$. Then

$$\sum_{k=1}^{n} \left| \frac{1}{T_k} \sum_{j=0}^{k} t_{k-j}\, x_j - \frac{1}{T_{k-1}} \sum_{j=0}^{k-1} t_{k-j-1}\, x_j \right| = 0$$

which says that $(N^t x) \in bv$. This completes the proof.

Theorem 2.6. The inclusion $bv(N^t) \subset c(N^t)$ strictly hold.

Proof. Since the inclusion $bv \subset c$ satisfy, the proof can be shown easily. So we omit the details.

3. $\alpha-, \beta-, \gamma-$ Dual of the Space $bv(N^t)$

In this section I construct the $\alpha-, \beta-, \gamma-$ dual of the space $bv(N^t)$. I start with some needed and important lemmas whose some parts related with the characterization of matrix transformations on the space $bv(N^t)$.

Lemma 3.1. [3] The infinite matrix $A \in (bv: \ell_1)$ if and only if

$$\sup_{l \in \mathbb{N}} \sum_{n} \left| \sum_{k=l}^{\infty} a_{nk} \right| < \infty \tag{3.1}$$

Lemma 3.2. [3] The infinite matrix $A \in (bv: bs)$ if and only if

$$\sup_{m,l \in \mathbb{N}} \left| \sum_{n=0}^{m} \sum_{k=l}^{\infty} a_{nk} \right| < \infty \tag{3.2}$$

Lemma 3.3. (Stieglitz, M., & Tietz, H. (1977)) The infinite matrix $A \in (bv: cs)$ if and only if the condition (3.2) holds and

(3.3) $\sum_n a_{nk}$ convergent for each $k \in \mathbb{N}$.

(3.4) $\sum_n \sum_k a_{nk}$ convergent.

Theorem 3.4. The $\alpha-$dual of the space $bv(N^t)$ is the set

$$d_1 = \left\{ a = (a_k) \in \omega : \sup_{l \in \mathbb{N}} \sum_{n} \left| \sum_{k=l}^{n-1} (-1)^{n-k-1} D_{n-k-1} T_{n-1} a_n + T_n a_n \right| < \infty \right\}$$

Proof. Let define the matrix $D = (d_{nk})$ via $a = (a_k) \in \omega$ by

$$d_{nk} = \begin{cases} (-1)^{n-k-1} D_{n-k-1} T_{n-1} a_n, & 1 \le k < n \\ T_n a_n, & k = n \\ 0, & k > n \end{cases}$$

for all $k, n \in \mathbb{N}$. Since the equality (2.2) holds then

(3.5) $$a_n x_n = \sum_{k=l}^{n-1}\left((-1)^{n-k-1} D_{n-k-1} T_{n-1} a_n\ y_k + T_n a_n y_n\right) = (Dy)_n$$

for all $n \in \mathbb{N}$. So by the relation (3.5) we may say that $ax = (a_n x_n) \in \ell_1$ whenever $x = (x_k) \in bv(N^t)$ if and only if $Dy \in \ell_1$ whenever $y = (y_k) \in bv$. This consequence gives us that $a \in \{(bv(N^t))^\alpha\}$ if and only if $D \in (bv: \ell_1)$. By the condition (3.1) of the Lemma 3.1 we can say that

$$\sup_{l\in\mathbb{N}} \sum_n \left| \sum_{k=l}^{n-1} (-1)^{n-k-1} D_{n-k-1} T_{n-1} a_n \ + T_n a_n \right| < \infty$$

which leads us to the set d_1 as the alpha-dual of the space $bv(N^t)$. This completes the proof.

Theorem 3.5. Let define the sets d_2 and d_3 as

$$d_2 = \left\{a = (a_k) \in \omega:\ \lim_n d_{nk}\ exists\ for\ each\ \ k \in \mathbb{N}\right\}$$

$$d_3 = \{a = (a_k) \in \omega:\ \sup_k \sum_n d_{nk}\ \}$$

where the matrix $D = (d_{nk})$ is defined as in Theorem 3.4. Then $\beta -$ dual of the space $bv(N^t)$ is the set $d_2 \cap d_3$.

Proof. This is similar to the proof of Theorem 3.4. with the conditions (4.1) and (4.3) of the Lemma 4.1.(iii) instead of the Lemma 3.1. So we omit the Details.

Theorem 3.6. $\gamma -$ dual of the space $bv(N^t)$ is the set d_3.

Proof. This is similar to the proof of Theorem 3.4. with the condition (4.1) of the Lemma 4.1.(i) instead of the Lemma 3.1. So we omit the Details.

4. Matrix Transformations Related to the Spaces $bv(N^t)$

In this present section, firstly I give some needed and related lemmas for the proof of theorems. Then I characterize some new matrix classes related with the sequence space $bv(N^t)$ and give some related results.

Lemma 4.1. Let $A = (a_{nk})$ be an infinite matrix. Then the following statements hold:

i) $A \in (\ell_1: \ell_\infty)$ if and only if

(4.1) $$\sup_{k,n\in\mathbb{N}} |a_{nk}| < \infty$$

ii) $A \in (\ell_1: \ell_1)$ if and only if

(4.2) $$\sup_{k\in\mathbb{N}} \sum_n |a_{nk}| < \infty$$

iii) $A \in (\ell_1: c)$ if and only if (4.1) holds, and there exists $a_k \in \mathbb{C}$ such that

(4.3) $$\lim_{n\to\infty} a_{nk} = a_k$$

iv) $A \in (c:\ell_1)$ if and only if

$$\sup_K \sum_n \left|\sum\nolimits_k a_{nk}\right| < \infty \tag{4.4}$$

where K is a finite subset of $\mathbb{N}$.

Theorem 4.1. Suppose that the matrices $A = (a_{nk})$ and $B = (b_{nk})$ connected with the relation

$$b_{nk} = \sum_{j=k}^{\infty} (-1)^{j-k} D_{j-k} T_k \, a_{nj} \tag{4.5}$$

for all $k, n \in \mathbb{N}$ and $\mu = \{\ell_1, c\ \ell_\infty\}$. Then $A \in (bv(N^t):\mu)$ if and only if $\{a_{nk}\}_{k\in\mathbb{N}} \in \{bv(N^t)\}^\beta$ for all $n \in \mathbb{N}$ and $B \in (bv:\mu)$.

Proof. Suppose that $A \in (bv(N^t):\mu)$ and $x = (x_k) \in bv(N^t)$. By the following equality having from the n^{th}-partial sum of the series $\sum_k a_{nk}x_k$ we have that

$$\sum_{k=0}^{n} a_{nk}x_k = \sum_{k=0}^{n}\sum_{j=k}^{n} (-1)^{j-k} D_{j-k} T_k \, a_{nj} y_k = \sum_{k=0}^{n} b_{nk} y_k$$

for all $k, n \in \mathbb{N}$. When Ax exists and belongs to the space μ for all $x \in bv(N^t)$, then By exists and belongs to the space μ for all $y \in bv$ after taking limit as $n \to \infty$**.** So as a consequence we have that $B \in (bv:\mu)$.

Conversely, suppose that $\{a_{nk}\}_{k\in\mathbb{N}} \in \{bv(N^t)\}^\beta$ for each $n \in \mathbb{N}$ and let us take a sequence $x = (x_k) \in bv(N^t)$. Then Ax exists and we may obtain from the following equality

$$\sum_{k=0}^{n} a_{nk}x_k = \sum_{k=0}^{n}\sum_{j=k}^{n} (-1)^{j-k} D_{j-k} T_k \, a_{nj} y_k = \sum_{k=0}^{n} b_{nk} y_k$$

as $n \to \infty$ that $Ax = By$ and since $By \in \mu$ for all $y \in bv$, then $Ax \in \mu$ for all $x \in bv(N^t)$. This last result says us that $A \in (bv(N^t):\mu)$. This completes the proof.

Theorem 4.2. Suppose that the matrices $A = (a_{nk})$ and $B = (b_{nk})$ connected with the relation

$$b_{nk} = \sum_{j=0}^{n} \frac{t_{n-j}}{T_n} a_{jk} \tag{4.6}$$

for all $k, n \in \mathbb{N}$. Then $A \in (\mathrm{c}: bv(N^t))$ if and only if

$$\sup_K \sum_n \left|\sum_{k=0}^{n}\sum_{j=0}^{n} \frac{t_{n-j}}{T_n} a_{jk}\right| < \infty \tag{4.7}$$

where K is a finite subset of $\mathbb{N}$.

Proof. Suppose that the matrix $A \in (\mathrm{c}: bv(N^t))$ and $x = (x_k) \in c$. Consider the following equality that

$$\sum_{j=0}^{n} \frac{t_{n-j}}{T_n} \sum_{k=0}^{j} a_{jk} x_{k=} \sum_{k=0}^{n} b_{nk} x_k$$

for all $k, n \in \mathbb{N}$. After taking limit as as $n \to \infty$ we have that $\{N^t(Ax)\}_n = (Bx)_n$ for all $n \in \mathbb{N}$. Since $Ax \in bv(N^t)$, $N^t(Ax) = Bx \in \ell_1$ says $B \in (c: \ell_1)$. Therefore, the condition of Lemma 4.1(iv) satisfies. That is

$$\sup_{K} \sum_{n} \left| \sum_{k=0}^{n} \sum_{j=0}^{n} \frac{t_{n-j}}{T_n} a_{jk} \right| < \infty$$

where K is a finite subset of $\mathbb{N}$.

Conversely, suppose that the condition (4.7) hold and the matrices $A = (a_{nk})$ and $B = (b_{nk})$ connected with the relation (4.6). Let us take a sequence $x = (x_k)$ from the sequence space c. Since $x \in c$ there is a positive real number M such that $\sup_{k \in \mathbb{N}} |x_k| < M$. Thus, one can derived from the following inequality that

$$\sum_{k=1}^{n} |\{N^t(Ax)\}_k - \{N^t(Ax)\}_{k-1}| = \sum_{k=1}^{n} \left| \sum_{k=0}^{n} \sum_{j=0}^{n} \frac{t_{n-j}}{T_n} a_{jk} x_j \right|$$

$$\leq M \sum_{k=1}^{n} \left| \sum_{k=0}^{n} \sum_{j=0}^{n} \frac{t_{n-j}}{T_n} a_{jk} \right| < \infty$$

we may say that $Ax \in bv(N^t)$ for all $x \in c$. So the infinite matrix $A \in (\mathrm{c}: bv(N^t))$. This completes the proof.

Corollary 4.3. Let $A = (a_{nk})$ be an infinite matrix and connected with $B = (b_{nk})$ by the relation (4.5). Then the following statements hold.

- **i)** The infinite matrix $A \in (bv(N^t): bs)$ if and only if the condition (3.2) hold with b_{nk} instead of a_{nk}.
- **ii)** The infinite matrix $A \in (bv(N^t): cs)$ if and only if the conditions (3.2), (3.3) and (3.4) hold with b_{nk} instead of a_{nk}.
- **iii)** The infinite matrix $A \in (bv(N^t): c)$ if and only if (4.1) and (4.3) hold with b_{nk} instead of a_{nk}.
- **iv)** The infinite matrix $A \in (bv(N^t): \ell_1)$ if and only if (4.2) holds with b_{nk} instead of a_{nk}.

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6. Competing Interests

The author declares that there is no conflict of interest regarding the publication of this paper.

7. Author's Contributions

OT defined new sequence space derived by Nörlund matrix domain of bounded variation sequences and studied some topological properties. OT computed the duals of new this sequence space and characterized some new the matrix classes. In the last section, some studies were summarized and some open problems were given by OT. The author read and approved the final manuscript.

8. Article Information

Some of the results of this study was presented in the Fourth International Conference on Applied Science, Energy and Environment (ICASEE 2017, 20-21 May, Erbil, IRAQ)

9. Conclusion

In 1978, the domain of Nörlund matrix N^t in the classical sequence spaces ℓ_∞ and ℓ_p were introduced by Wang (1978). where $1 < p < 1$. In 1978, the domain of Cesaro matrix C_1 of order one in the classical sequence spaces ℓ_∞ and ℓ_p were introduced by Ng and Lee (1978) where $1 < p < 1$. Following Ng and Lee (1978) Sengönül and Başar (2005) have studied the domain of Cesaro matrix C_1 of order one in the classical sequence spaces c_0 and c. In order to fill up the gap in the existing literature Tuğ and Basar (2016) studied the matrix domain of Nörlund mean in the classical sequence spaces c_0 and c. Recently Yeşilkayagil and Başar (2014), and Yeşilkayagil and Başar (2017) have studied the paranormed Nörlund sequence space of non-absolute type, and domain of Nörlund matrix in some Maddox's spaces. Moreover, Tuğ and Başar (2016) studied the matrix domain of Nörlund mean in the sequence spaces f and f_0 to fill the gap in the literature after the studies by Duran (1972) and Yeşilkayagil and Başar (2015).

Kirisci (2014) studied the sequence space bv and its some applications with some special matrices. In this study, I tried to fill up the gap in the existing literature of the sequence space bv and its Nörlund matrix domain by calculating some topological properties, the $\alpha-, \beta-, \gamma-$ duals and charactarizing some matrix transformations in/on the sequence space $bv(N^t)$. Following matrix classes $\left(f: bv(N^t)\right)$ and $(bv(N^t): f)$ are still open problems in order to characterize and calculate the matrix transforms

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