Nörlund Matrix Domain on Sequence Spaces of p-adic Numbers

Orhan Tuğ[1]

[1] Mathematics Education Department, Ishik University, Erbil, Iraq
Correspondence: Orhan Tug, Mathematics Education Department, Ishik University, Erbil, Iraq.
E-mail: orhan.tug@ishik.edu.iq

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Abstract: In this paper, we introduce some new sequence spaces p-adic numbers $l_\infty{}^{(p)}(N^t)$, $c^{(p)}(N^t)$ and $c_0{}^{(p)}(N^t)$ as Nörlund matrix domain in the sequence spaces $l_\infty{}^{(p)}$, $c^{(p)}$ and $c_0{}^{(p)}$, respectively. Moreover, $\alpha-$, $\beta-$ and $\gamma-$ dual of these new spaces are calculated with some topological properties. We characterize some new matrix classes related with the spaces $l_\infty{}^{(p)}(N^t)$, $c^{(p)}(N^t)$ and $c_0{}^{(p)}(N^t)$ and we conclude the paper with some significant results and an application.

Keywords: Nörlund Matrix, Sequence Spaces of p-adic Numbers, Matrix Transformations

1. Introduction

In the real case, we denote the space of all real valued sequences by ω. Each vector subspace of ω is called as a sequence space as well. The spaces of all bounded, convergent and null sequences are denoted by ℓ_∞, c and c_0, respectively. By ℓ_1, ℓ_p, cs, cs_0 and bs, we denote the spaces of all absolutely convergent, p-absolutely convergent, convergent, convergent to zero and bounded series, respectively; where $1 < p < \infty$.

A linear topological space λ is called a K-space if each of the map $\rho_i: \lambda \longrightarrow \mathbb{C}$ defined by $\mathrm{p_i(x) = x_i}$ is continuous for all $i \in \mathbb{N}$, where $\mathbb{C}$ denotes the complex field and $\mathbb{N} = \{0,1,2,3,\dots\}$. A K-space λ is called an FK-space if λ is a complete linear metric space. If an FK-space has a normable topology then it is called a *BK*-space. If λ is an *FK*-space, $\Phi \subset \lambda$ and $\left(e^k\right)$ is a basis for λ then λ is said to have *AK* property, where $\left(e^k\right)$ is the sequence whose only non-zero term is a 1 in k^{th} place for each $\mathrm{k} \in \mathbb{N}$ and $\Phi = span\{e^k\}$. If Φ is dense in λ, then λ is called *AD*-space, thus *AK* implies *AD*.

Let λ and μ be two sequence spaces, and $A = (a_{nk})$ be an infinite matrix of real or complex numbers, where $\mathrm{n, k} \in \mathbb{N}$. For every sequence $x = (x_\mathrm{k}) \in \lambda$ the sequence $Ax = Ax = ((Ax)_\mathrm{n}) \in \mu$ is called A-transform of x, where

$$(1.1) \qquad (Ax)_\mathrm{n} = \sum_{k=0}^{\infty} a_{nk}\, x_k\,.$$

Then, A defines a matrix mapping from λ to μ and we show it by writing $A: \lambda \longrightarrow \mu$.

By $A \in (\lambda : \mu)$, we denote the class of all matrices A such that $A : \lambda \longrightarrow \mu$ if and only if the series on the right side of (1.1) converges for each $\mathrm{n} \in \mathbb{N}$ and every $\mathrm{x} \in \lambda$, and we have $Ax =$

$((Ax)_n)$ belongs to μ for all x ∈ λ. A sequence x is said to be A-summable to l and is called as the A-limit of x.

Let $\boldsymbol{\lambda}$ be a sequence space and A be an infinite matrix. The matrix domain λ_A of A in $\boldsymbol{\lambda}$ is defined by

$$\lambda_A = \{x = (x_k) \in \omega : Ax \in \lambda\}$$

which is a sequence space.

Let (t_k) be a nonnegative real sequence with $t_0 > 0$ and $T_n = \sum_{k=0}^{n} t_k$ for all $k, n \in \mathbb{N}$. Then, the Nörlund mean with respect to the sequence $t = (t_k)$ is defined by the matrix $N^t = (a_{nk}^t)$ as follows

(1.2)
$$a_{nk}^t = \begin{cases} \dfrac{t_{n-k}}{T_n} & , \quad 0 \le k \le n \\ 0 & , \quad k > n \end{cases}$$

For every $k, n \in \mathbb{N}$ it is known that the Nörlund matrix N^t is a Teoplitz matrix if and only if $\frac{t_n}{T_n} \longrightarrow 0, as\ n \longrightarrow \infty$. Furthermore, if we take $t = e = (1,1,1,\dots)$, then the Nörlund matrix N^t is reduced to Cesàro mean C_1 of order one and if we choose $t_n = A_n^{r-1}$ for every $n \in \mathbb{N}$, then the N^t Nörlund mean becomes Cesàro mean C_r of order r, where $r > -1$ and

$$A_n^t = \begin{cases} \dfrac{(r+1)(r+2)\dots(r+n)}{n!} & , \quad n = 1,2,3,\dots \\ 0 & , \quad n = 0 \end{cases}$$

Let $t_0 = D_0 = 1$ and define D_n for $n \in \{1,2,3,\dots\}$ by

(1.3)
$$D_n = \begin{vmatrix} t_1 & 1 & 0 & 0 & \cdots & 0 \\ t_2 & t_1 & 1 & 0 & \cdots & 0 \\ t_3 & t_2 & t_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ t_{n-1} & t_{n-2} & t_{n-3} & t_{n-4} & \cdots & 1 \\ t_n & t_{n-1} & t_{n-2} & t_{n-3} & \dots & t_1 \end{vmatrix}$$

With $D_1 = t_1$, $D_2 = (t_1)^2 - t_2$, $D_3 = (t_3)^3 - 2t_1t_2 + t_3 \dots \dots$ then the inverse matrix $U^t = (u_{nk}^t)$ of Nörlund matrix N^t was defined by Mears in [2] for all n ∈ ℕ as follows

(1.4)
$$u_{nk}^t = \begin{cases} (-1)^{n-k} D_{n-k} T_k & , \quad 0 \le k \le n, \\ 0 & , \quad k > n. \end{cases}$$

2. P-adic Numbers

We begin with the definitions of p-adic numbers and p-adic integers with some topological properties.

Definition 2.1. (Katok, S. (2007)) In what follows p is a fixed prime number. The set $\mathbb{Q}_p$ is a completion of the rational numbers $\mathbb{Q}$ with respect to the norm $|\cdot|_p : \mathbb{Q} \to \mathbb{R}$ given by

(2.1.1)
$$|x|_p = \begin{cases} p^{-r} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$$

where $x = p^r \frac{m}{n}$, $\forall x \in \mathbb{Q}$ and $m \in \mathbb{Z}$ and $n \in \mathbb{N}^+$ s.t. $(p,m) = (p,n) = 1$. The absolute value $|\cdot|_p$ is named non-Archimedean and the most important and useful property of this absolute value is satisfying the following inequality which is called "strong triangle inequality."

(2.1.2) $$|x+y|_p \le \max\{|x|_p, |y|_p\}$$

i.e. if $|x|_p > |y|_p$ then $|x+y|_p = |x|_p$. This property is the most crucial property of non-Archimedean metric. Any p-adic number $x \in \mathbb{Q}_p$, where $x \neq 0$ is uniquely represented in the form

(2.1.3) $$x = p^r(x_0 + x_1 p^1 + x_2 p^2 + \cdots)$$

where $r \in \mathbb{Z}$ and x_i are integers, $0 \le x_i < p,\ x_0 > 0, i = 0,1,2, \ldots.$ This form is called the canonic form of $x \in \mathbb{Q}_p$ and $|x|_p = p^{-r}$.

Definition 2.2. (Sally, P. J. (1998)) Let $x \in \mathbb{Q}_p$ be a p-adic number. Then the following set is called the p-adic integers.

(2.2.1) $$\mathbb{Z}_p = \{x \in \mathbb{Q}_p : |x|_p \le 1\}$$

For each $c \in \mathbb{Q}_p$, and $r > 0$, $B(c,r) = \{x \in \mathbb{Q}_p : |x - c|_p \le r\}$ is called the p-adic ball.

The spaces $bs^{(p)}$, $cs^{(p)}$ and $l_1^{(p)}$of p-adic numbers were defined as the series space of p-adic numbers whose sequences of the partial sum is bounded, the series space of p-adic numbers whose sequences of the partial sum is convergent, and the space of absolutely summable series of p-adic numbers, respectively. Those spaces are complete metrizable topological vector spaces of p-adic numbers with respect to the p-adic norm defined by (2.1.1).

It can be obtained from here that all the series $\sum_k x_k$ converges if and only if $x_k \to 0\ as\ k \to \infty$. Therefore, $c_0^{(p)}$ coincides with the space of all convergent series $cs^{(p)}$.

Let μ be any sequence space of p-adic numbers. Then the duals of any sequence space in p-adic numbers defined by

$$\{\mu^{(p)}\}^{\alpha} = \{a = (a_k) \in w^{(p)} : ax = (a_k x_k) \in\ l_1^{(p)}, \quad for\ all\ x = (x_k) \in \mu\}$$

$$\{\mu^{(p)}\}^{\beta} = \{a = (a_k) \in w^{(p)} : ax = (a_k x_k) \in\ cs^{(p)}, \quad for\ all\ x = (x_k) \in \mu\}$$

$$\{\mu^{(p)}\}^{\gamma} = \{a = (a_k) \in w^{(p)} : ax = (a_k x_k) \in\ bs^{(p)}, \quad for\ all\ x = (x_k) \in \mu\}$$

Definition 2.3. (Andree, R. V., & Petersen, G. M. (1956)**)** A sequence $s = (s_n)$ obtained from the infinite matrix $A = (a_{mn})$ and the sequence $x = (x_n)$ using the relation $s_n = \sum_{k=0}^{n} a_{mk}\, x_k$ is called $A = (a_{mn})$ transform of $x = (x_n)$. If $s = (s_n)$ converges to T, the matrix $A = (a_{mn})$ is said to sum the sequence $x = (x_n)$ to the sum T.

Definition 2.4. (Andree, R. V., & Petersen, G. M. (1956)**)** The method of summation defined by the matrix $A = (a_{mn})$ is called regular in the p-adic field $\mathbb{Q}_p$ if every convergent sequence $x = (x_n)$ is equal to its transform $s = (s_n)$ in $\mathbb{Q}_p$. The sequences $x = (x_n)$ and $s = (s_n)$ are equal if $\lim_{n\to\infty}|x_n - s_n|_p = 0$. Clearly if $x = (x_n)$ is p-convergent, then $s = (s_n)$ is p-convergent. The inverse does not need to be held.

Theorem 2.5. (Andree, R. V., & Petersen, G. M. (1956)**)** The matrix $A = (a_{mn})$ is called p-regular if and only if the following conditions are held.

(2.5.1) $$\lim_{m\to\infty} |a_{mn}|_p = 0$$

(2.5.2) $$\lim_{n\to\infty} \left|\sum_{k=0}^{n} a_{mk}\, x_k - 1\right|_p = 0$$

(2.5.3) $$|a_{mn}|_p \le M$$

The method $s_n = \sum_{k=0}^{n} a_{mk}\, x_k$ is also called p-regular.

3. Some New Sequence Spaces of p-adic Numbers

In this section, we define some new sequence spaces of p-adic numbers as Nörlund matrix domain in the sequence spaces of all bounded, convergent and null sequences of p-adic numbers, respectively.

Now we may define the following new sequence spaces,

$$l_\infty{}^{(p)}(N^t) = \left\{x = (x_k) \in w^{(p)} : \sup_{n\in\mathbb{N}} \left|\sum_{k=0}^{n} \frac{t_{n-k}}{T_n} x_k\right|_p < \infty\right\},$$

$$c^{(p)}(N^t) = \left\{x = (x_k) \in w^{(p)} : \exists l \in \mathbb{Q}_p, \ni \lim_{n\to\infty} \left|\sum_{k=0}^{n} \frac{t_{n-k}}{T_n} x_k - l\right|_p = 0\right\},$$

$$c_0^{(p)}(N^t) = \left\{x = (x_k) \in w^{(p)} : \exists l \in \mathbb{Q}_p, \ni \lim_{n\to\infty} \left|\sum_{k=0}^{n} \frac{t_{n-k}}{T_n} x_k\right|_p = 0\right\},$$

Theorem 3.1. The sequence spaces $c^{(p)}(N^t)$ and $c_0^{(p)}(N^t)$ are the linear Banach space with the p-adic norm $\|(N^t x)\|_p, = (\|x\|)_{c^{(p)}(N^t)}$.

Proof: The linearity of the space $c^{(p)}(N^t)$ is clear. By Wilansky, A. (2000), we can easily say that the space $c^{(p)}(N^t)$ is BK- space since the Nörlund matrix is a triangle and the space $c^{(p)}$ is BK-space. It is well known that every Cauchy sequence is convergent in the p-adic number field. Moreover, the space $c^{(p)}$ is complete normed space with the norm $(\|x\|)_{c^{(p)}(N^t)}$, then it is easy to show that every Cauchy sequence in $c^{(p)}(N^t)$ converges to any number in $c^{(p)}(N^t)$. This completes the proof.

Corollary 3.2. The sequence space $l_\infty{}^{(p)}(N^t)$ is complete normed space with the supremum p-norm.

Theorem 3.3. The conclusion $c_0^{(p)}(N^t) \subset c^{(p)}(N^t)$ strictly holds.

Proof: Suppose that $x = (x_k) \in c_0^{(p)}(N^t)$. Then, $N^t x \in c_0^{(p)}$. It is known that $c_0^{(p)} \subset c^{(p)}$, then $N^t x \in c^{(p)}$ and as a result $x \in c^{(p)}(N^t)$ means that the inclusion holds. Now we should show that the inclusion is strict. For this, we must take a sequence which belongs to $c^{(p)}(N^t)$ but not in $c_0^{(p)}(N^t)$. Let decide the sequence $x_k = e_k$, for all $k \in \mathbb{N}$ whch belongs to $\mathbb{Q}_p$. Then the N^t transform of x equals to $(N^t x)_n = 1$ which says that $N^t x \in c^{(p)}$ but not belongs to $c_0^{(p)}$. This completes the proof.

Theorem 3.4. The conclusion $c^{(p)} \subset c^{(p)}(N^t)$ strictly holds.

Proof: Suppose that $x = (x_n) \in c^{(p)}$ then $\lim_{n\to\infty} |x_n - l|_p = 0$ holds for any $l \in \mathbb{Q}_p$. Then $x_n = \sum_{k=0}^{n} (-1)^{n-k} D_{n-k} T_k y_k$ and

$$\lim_{n\to\infty} |x_n|_p = \lim_{n\to\infty} \left| \sum_{k=0}^{n} (-1)^{n-k} D_{n-k} T_k y_k \right|_p$$

$$\leq \lim_{n\to\infty} \left(\max_n \{ |(-1)^n D_n T_0 y_0|_p, \ldots, |(-1)^0 D_0 T_n y_n|_p \} \right)$$

$$\leq \lim_{n\to\infty} |T_n y_n|_p$$

means that $x = (x_n) \in c^{(p)}(N^t)$ which completes the proof.

Corollary 3.5. The conclusion $l_\infty{}^{(p)} \subset l_\infty{}^{(p)}(N^t)$ strictly holds.

4. $\alpha-, \beta-$ *and* $\gamma-$Dual of the Sequence Spaces of p-adic Numbers

In this present section, we state the $\alpha-, \beta-$ *and* $\gamma-$dual of the new sequence spaces. We start with some significant Lemmas which will be used in this present and the following chapters.

Lemma 4.1. Let $A = (a_{nk})$ be an infinite matrix. Then $A \in (c^{(p)} : l_1^{(p)})$ if and only if

$$\sup_{K\in F} \sum_n \left| \sum_{k\in K} a_{nk} \right|_p < \infty \tag{4.1.1}$$

where F is finite subset of $\mathbb{N}$.

Lemma 4.2. Let $A = (a_{nk})$ be an infinite matrix. Then $A \in (c^{(p)} : l_\infty^{(p)})$ if and only if

$$\sup_{n\in\mathbb{N}} \sum_k |a_{nk}|_p < \infty \tag{4.2.1}$$

Lemma 4.3. (Monna, A. F. (1970)) Let $A = (a_{nk})$ be an infinite matrix. Then $A \in (c^{(p)} : c^{(p)})$ if and only if

(4.3.1) $\lim_{n,k\in\mathbb{N}} |a_{nk}|_p < \infty$

(4.3.2) $\exists a_k \in \mathbb{Q}_p \textit{ such that } \lim_{n\to\infty} |a_{nk} - a_k|_p = 0, \ \textit{for all } k \in \mathbb{N}.$

(4.3.3) $\exists a \in \mathbb{Q}_p \textit{ such that } \lim_{n\to\infty} \left|\sum_k a_{nk} - a\right|_p = 0.$

Theorem 4.4. The α −dual of the spaces $c^{(p)}(N^t)$ and $c_0^{(p)}(N^t)$ is the set

$$\alpha_p^t := \left\{a = a_k \in w_p : \sup_{K\in F} \sum_n \left|\sum\nolimits_{k\in K} (-1)^{n-k} D_{n-k} T_k a_n\right|_p < \infty\right\}$$

Proof: Suppose that the sequence $a = (a_k) \in w_p$ and $x = (x_k) \in c^{(p)}(N^t)$ then the following equality

$$a_n x_n = \sum_{k=0}^{n} (-1)^{n-k} D_{n-k} T_k a_n y_k = (By)_n$$

holds for all $k, n \in \mathbb{N}$ whenever $y = (y_k) \in c^{(p)}$ since the relation $x_k = (U^t y)_k$ satisfied. If we define the matrix $B = (b_{nk})$ as Tuğ & Başar (2016) defined, then we may say that $ax = (a_n x_n) \in l_1^{(p)}$ whenever $x = (x_k) \in c^{(p)}(N^t)$ if and only if $By \in l_1^{(p)}$ whenever $y = (y_k) \in c^{(p)}$. Thus, the matrix $B \in (c^{(p)} : l_\infty^{(p)})$ and the condition of Lemma 4.1. holds with b_{nk} instead of a_{nk}, that is,

$$\sup_{K\in F} \sum_n \left|\sum\nolimits_{k\in K} (-1)^{n-k} D_{n-k} T_k a_n\right|_p < \infty$$

which gives the results as desired that $\{c^{(p)}(N^t)\}^\alpha = \alpha_p^t$.

Since the proof of $\beta - and\, \gamma -$ dual of $c^{(p)}(N^t)$ and $c_0^{(p)}(N^t)$ can be shown with the same method used in Theorem 4.4. with the Lemma 4.3. and Lemma 4.2., respectively. So we give the following corollaries.

Corollary 4.5. The β −dual of the spaces $c^{(p)}(N^t)$ and $c_0^{(p)}(N^t)$ is the set $\beta_p^t \cap cs_p$ where the set β_p^t is defined by

$$\beta_p^t := \left\{a = a_k \in w_p : \sup_{n\in\mathbb{N}} \sum_k \left|\sum_{j=k}^{n} (-1)^{j-k} D_{j-k} T_k a_j\right|_p < \infty\right\}$$

Corollary 4.6. The γ −dual of the spaces $c^{(p)}(N^t)$ and $c_0^{(p)}(N^t)$ is the set β_p^t.

5. Matrix Transformations Related to the Sequence Spaces $c^{(p)}(N^t)$, $c_0^{(p)}(N^t)$ and $l_\infty{}^{(p)}(N^t)$

In this section we characterize some new matrix classes from the new sequence spaces $c^{(p)}(N^t)$, $c_0^{(p)}(N^t)$ and $l_\infty{}^{(p)}(N^t)$ into the sequence spaces of p-adic numbers $l_\infty{}^{(p)}$, $c^{(p)}$ and $c_0{}^{(p)}$ and vice versa. Throughout the section, we use the relation between the matrices $A=(a_{nk})$ and $F=(f_{nk})$, and $G=(g_{nk})$ which are defined and studied by Tuğ & Başar (2016) as follow,

$$f_{nk}=\sum_{j=k}^{\infty}(-1)^{j-k}D_{j-k}T_k a_{nj} \qquad and \qquad g_{nk}=\sum_{j=k}^{n}\frac{t_{n-k}}{T_n}a_{jk}\ , \quad for\ all\ k,l \in \mathbb{N}. \tag{5.1}$$

Theorem 5.1. $A\in(c^{(p)}(N^t):l_\infty^{(p)})$ if and only if $A_n\in\{c^{(p)}(N^t)\}^\beta$ for each $n\in\mathbb{N}$ and $F\in(c^{(p)}:l_\infty^{(p)})$.

Proof: Let suppose that $A\in(c^{(p)}(N^t):l_\infty^{(p)})$ and $=(x_k)\in c^{(p)}(N^t)$. Then it can be said that Ax exists and belongs to $l_\infty^{(p)}$. Then the m^th partial sum of A- transform of x,

$$\sum_{k=0}^{m}a_{nk}x_k=\sum_{k=0}^{m}\sum_{j=k}^{m}(-1)^{j-k}D_{j-k}T_k a_{nj}y_k$$

for all $m,n\in\mathbb{N}$. When we pass to limit as $m\to\infty$, we have that

$$\sum_{k}a_{nk}x_k=\sum_{k}\sum_{j=k}^{\infty}(-1)^{j-k}D_{j-k}T_k a_{nj}y_k$$

for all $n\in\mathbb{N}$ which means that $Ax=Fy\in l_\infty^{(p)}$ whenever $y=(y_k)\in c^{(p)}$. So $F\in(c^{(p)}:l_\infty^{(p)})$ as we desired.

Theorem 5.2. $A\in(c^{(p)}:l_\infty^{(p)}(N^t))$ if and only if $G\in(c^{(p)}:l_\infty^{(p)})$.

Proof: The proof can be shown by like method used in Theorem 5.1. with the relation (5.1) between $A=(a_{nk})$ and $G=(g_{nk})$.

Corollary: 5.3. Let $A=(a_{nk})$ be an infinite matrix. Then, the following statements hold:

i) $A\in(c^{(p)}(N^t):c^{(p)})$ if and only if the conditions (4.3.1)-(4.3.3) hold with f_{nk} instead of a_{nk}.
ii) $A\in(c^{(p)}:c^{(p)}(N^t))$ if and only if the conditions (4.3.1)-(4.3.3) hold with g_{nk} instead of a_{nk}.
iii) $A\in(c^{(p)}(N^t):l_1^{(p)})$ if and only if the condition (4.1.1) holds with f_{nk} instead of a_{nk}.
iv) $A\in(c^{(p)}:l_1^{(p)}(N^t))$ if and only if the condition (4.1.1) holds with g_{nk} instead of a_{nk}.

6. An Application on the Sequence Spaces $c^{(p)}(N^t)$, $c_0^{(p)}(N^t)$ and $l_\infty{}^{(p)}(N^t)$

In this section we define a Nörlund type matrix in p-adic form and we show some application on sequence spaces of p-adic numbers.

Suppose that $t_k = \left(\frac{1}{p}\right)^k$ be a nonnegative p-adic sequence where p is prime numbers and $T_n = \sum_{k=0}^{n} \left(\frac{1}{p}\right)^k$ for all $k, n \in \mathbb{N}$. Then, the Nörlund type mean with respect to the sequence $t = (t_k)$ is defined by the matrix $N^{(p)} = \left(a_{nk}^{(p)}\right)$ as follows

(6.1)
$$a_{nk}^{(p)} = \begin{cases} \dfrac{p-1}{p^{n-k}} \ , & 0 \le k \le n \\ 0 \ , & k > n \end{cases}$$

for every $k, n \in \mathbb{N}$. It is know that the Nörlund type matrix $N^{(p)}$ is a Teoplitz matrix if and only if $\frac{p-1}{p^{n-k}} \longrightarrow 0,\ as\ \ n \longrightarrow \infty$. The inverse of Nörlund type matrix $N^{(p)}$ is $\left(N^{(p)}\right)^{-1} = U^{(p)} = \left(u_{nk}^{(p)}\right)$ defined as follow

$$u_{nk}^{(p)} = \begin{cases} (-1)^{n-k} \left(\dfrac{1}{p}\right)^{n-k} T_n, & n-1 \le k \le n \\ 0 \ , & otherwise \end{cases}$$

So all above theorems and corollaries are satisfied with the Nörlund type matrix $N^{(p)} = \left(a_{nk}^{(p)}\right)$. The results which are given above are more general with the Nörlund matrix rather than Nörlund type matrix $N^{(p)} = \left(a_{nk}^{(p)}\right)$.

7. Conclusion

The concept of p-adic numbers has been studied by many of the mathematicians as p-adic algebra and as p-adic analysis. Most of the important topological and algebraic properties were discovered and introduced. Nowadays, there are significant applications of p-adic numbers and p-adic analysis in pure mathematics, mathematical physics, applied statistics and computer sciences.

The Nörlund matrix domain on null and convergent sequences, and almost convergent sequences spaces were studied by Tuğ and Başar (2016). They investigated some topological properties, duals and some important matrix transformations.

In this paper, we calculated the Nörlund domain on the sequence spaces ${l_\infty}^{(p)}$, $c^{(p)}$ and ${c_0}^{(p)}$ of p-adic numbers and we proved some topological properties of the new sequence spaces of p-adic numbers. Then some important inclusion relations were proved. Moreover, the $\alpha-, \beta-$ and $\gamma-$ duals were calculated. In the last section we characterized some new matrix classes related with new sequence spaces ${l_\infty}^{(p)}(N^t)$, $c^{(p)}(N^t)$ and ${c_0}^{(p)}(N^t)$ of p-adic numbers and an application of Nörlund type matrix was stated.

References

Andree, R. V., & Petersen, G. M. (1956). Matrix methods of summation, regular for p-adic valuations. *Proceedings of the American Mathematical Society*, *7*(2), 250-253.
Bachman, G. (1964). *Introduction to p-adic Numbers and Valuation Theory. Academic Press. Inc.*
Borwein, D., & Jakimovski, A. (1994). Matrix transformations of power series. *Proceedings of the*

American Mathematical Society, *122*(2), 511-523.

Cho, I. (2014). p-adic Banach space operators and adelic Banach space operators. *Opuscula Mathematica*, *34*.

Gouvêa, F. Q. (1997). Elementary Analysis in ℚp. In *p-adic Numbers* (pp. 87-132). Springer Berlin Heidelberg.

Katok, S. (2007). *p-adic Analysis Compared with Real* (Vol. 37). American Mathematical Soc.

Mahler, K. (1973). *Introduction to P-Adic Numbers and Their Function*. CUP Archive.

Robert, A. M. (2000). A course in p-adic analysis, volume 198 of Graduate Texts in Mathematics.

Sally, P. J. (1998). An introduction to p-adic fields, harmonic analysis and the representation theory of SL2. *Letters in Mathematical Physics*, *46*(1), 1-47.

Tug, O., & Basar, F. (2016). On the domain of norlund mean in the spaces of null and convergent sequences, TWMS J. *Pure Appl. Math*, *7*(1), 76-87.

Tuğ, O., & Başar, F. (2016). On the spaces of nörlund almost null and nörlund almost convergent sequences. *Filomat*, *30*(3), 773-783.

Wilansky, A. (2000). *Summability through Functional Analysis* (Vol. 85). Elsevier.