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Nonlinear structural analysis technique based on flexibility method by Pade approximants

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ABSTRACT

This work proposes an improved numerical methodology based on the flexibility method to study the geometric nonlinearity of space cable structures. The proposed approach makes use of the Pade approximation to enhance the performance of computation. The transformation to the Pade arrangement is particularly successful in quickly speeding up convergence and obtaining the solution when working with complex structures that demonstrate geometrically nonlinear properties. In contrast to previous approaches, the suggested method directly solves the problem by formulating an algebraic system of nonlinear equations using the Pade approximation. To arrive at an analytical solution, some of the most well-established methods that make use of iterative techniques include dynamic relaxation, finite element analysis, and minimum total potential energy. A comprehensive evaluation of the proposed technique's precision and reliability was conducted using six different numerical examples. The recommended method's accuracy, consistency, and computational efficiency are shown by carefully comparing the results with those of techniques that have been around for a long time. This work contributes to the advancement of numerical approaches for the analysis of complex structural behavior by providing a reliable and efficient alternative. Moreover, this work is beneficial for both academics and professionals working in the field.

Keywords: Cable Net Structure; Spatial Structure; Geometric Nonlinearity; Static Analysis; Force Method; Nonlinear Analysis.

1. Introduction

Sources of nonlinearity in structures can be classified into three categories: material nonlinearity, boundary nonlinearity, and geometric nonlinearity. Currently, large space structures are requested, mainly in which the cable member structures are the main element in assembly. Cables provide interesting perspectives to form attractive spatial grid structures with high flexibility. It is noteworthy that cable nets show great structural flexibility and nonlinear response under loading conditions. However, the most challenging aspect of cable structure analysis is the absence of flexural rigidity, which results in high displacements. Consequently, geometric nonlinearity is required to be considered in the analysis of cable structures. The geometric nonlinearities originate when the structural deformation is experiencing a noticeable strain to make the cable's stress sufficient to produce a state of equilibrium in deformed states. The efficiency of cable structures depends on prestressing to attain a desirable appearance and function with the required stability. The inserted prestressing effort offers advances in structural inflexibility, the lessening of structural distortion, and the redistribution of internal stress, proffering a more cost-effective structure[1, 2]. As Kwan[3] stated, the behavior (initial stiffness) of cable nets depends on the prestressing rather than its axial stiffness. So, any improvement in finding new analysis techniques for such structures is demanded.

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Various recent methods for analyzing cable structures have been thoroughly reviewed[3-10]. Most recent methods insist on algorithmic procedures, computer operation aspects, and programming, which contribute to their prorated complexity. In contrast, the vital attention in deriving the proposed technique in this paper is the lucidity of the essential characteristics of cable structures. Therefore, the essential emphases of this paper are: i) to propose a new method for analyzing simple and complex cable structures under static loadings; and ii) to compare and evaluate this approach with several highly nonlinear structural problems.

Various recent methods for analyzing cable structures have been reviewed, which recently exist thoroughly[3-10], and are used for both the static and dynamic analysis of structural cables. The susceptibilities of these solution approaches are dissimilar from each other. Most of them are very complex and require sufficient experience to use. In this section, four of these popular approaches are described briefly.

1.1. Finite element approach

More than a few researchers worked on improving the flexibility and stiffness matrices to provide a closer nonlinearity behavior for the nonlinear geometrical structures. Abad et al.[11] found the tangent stiffness matrices of continuous and discrete cables that were subjected to static and thermal loads that were spread out and moved around. Utilizing a Gauss iteration pattern to propose a stiffness matrix for catenary cable, considering geometry and material nonlinearity, has been done by Naghavi Riabi and Shooshtari[7]. Crusells-Girona et al.[12] proposed a general finite element technique for the nonlinear analysis of cable structures depending on curvilinear coordinates by using a mixed variational formulation that used small numbers of finite elements for detecting nodal displacements and axial internal forces. Moreover, a different total Lagrangian finite element for geometric nonlinearity is set out by Coda et al.[13] to analyze cable nets and a wide-span suspended bridge.

Whenever it comes to nonlinear structural analysis, the finite element approach has a number of limitations, the most significant of which are its computing complexity and resource requirements. The production of meshes in finite element analysis requires a significant amount of processing power and time, which renders it inefficient for some applications, notably those that include structures that are both large and complex.

1.2. Dynamic relaxation method

The Dynamic Relaxation Method (DRM) is a nonlinear analysis method of structures that presents for analyzing nonlinear geometrical structures attractively [14]. Meanwhile, only an analytical framework can perform both form-finding and analysis[3]. The DRM is operated to make a solution for the ordinary and partial differential equations as an individual or a set of equations. The equilibrium equations are altered to take the form of a simulated dynamic system. Damping force and fictitious mass will be added, subsequently presented in a finite-difference formula, and solved via iterations[9, 15]. Reaching a steady state of equilibrium for the space dynamic equation expresses the result of the static equations and serves as the foundation for the dynamic relaxation technique[3, 9]. Damping coefficients, inertia mass, time step[16], and displacement[9] are the main criteria for controlling the speed, rate of convergence, and stability of the solution process. Rezaiee-Pajand and Hakkak[17] proposed a different procedure for dynamic relaxation by using Taylor series expansion. Based on utilizing the Gerschgorin circle theorem, Rezaiee-Pajand and Alamatian[18] suggested new damping and mass matrices. In the work performed by Hüttner et al.[19], several schemes of dynamic relaxation methods are proposed using viscous and kinetic damping. The cable elements are introduced as parabolic cable, catenary cable, and tensile bar elements concerning the mass distribution of the cables. In another study, Rezaiee-Pajand and Mohammadi-Khatami[9] used a dynamic relaxation scheme to generate six different stiffness matrices for the nonlinear geometric analysis of cables.

Dynamic relaxation for solving the problems of static nonlinear structural analysis can be attained only at the steady state of the structure. The computational time for solving and convergence is essentially determined by the three parameters: the matrix of fictitious mass, the damping coefficient, and the incremental time. Thus, it requires a longer duration to pass through the equilibrium path to approach the solution, particularly for complex systems. Therefore, the quicker solver technique will be more efficient for computation.

1.3. Linear and nonlinear force approach

The fundamental equations in this method are equilibrium, compatibility, and flexibility relations. Improving the linear force method has been a focus for many researchers. For instance, based on the principle of virtual work, Calladine[20] confirmed that the transpose of the equilibrium matrix is equal to the compatibility matrix. The force method is primarily applied in analyzing prestressed spatial structures with infinitesimal mechanisms, and experimental work has been completed by Pellegrino[21] to validate this approach. Later, Pellegrino[22] introduced the equilibrium matrix's singular value decomposition (SVD) to indicate the structural assemblies' static and kinematic nature relating to the physical properties in deriving the stress and displacement formulation.

This technique has been further enhanced as a nonlinear analysis approach for geometrically nonlinear structures. Kwan[3] reused the main classical equations of the force method and expressed the member actuation for a prestressed cable structure in terms of displacement using the Taylor series. In addition, Luo and Lu[23] extended the linear force method to analyze nonlinear geometric cable structures. They proposed an algorithm using SVD for the equilibrium matrix in every step of the iteration process. Xu and Luo[24], on the other hand, used the nonlinear force method to propose an iteration procedure for restoring the displaced joints and controlling the prestressed level of cable net systems. Similarly, Yuan et al.[25] used the nonlinear force method to control the stress and shape of the cable-strut structure. They utilized the Moore–Penrose pseudoinverse to compute the minimal necessary actuation. Furthermore, Manguri and Saeed[8], as well as Saeed, et al.[10], proposed an approximate linear force analysis technique. It is based on updating the joint coordinates of the structural geometry in every iteration for the discretized applied load.

Even though different solving algorithms have been used for the force method in previous studies, some things still lead to inaccurate results. For example, using the steady states of the self-stress matrix or the constant equilibrium matrix to come up with the analysis formulation is one of these things. Some studies, though, added geometric nonlinearity to compatibility and equilibrium matrices in the form of an iteration[26, 27]. So, we need a direct equation that can show both compatibility and equilibrium in the fully deformed shape as a set of algebraic nonlinear equations. This is what we did to come up with the proposed equation.

1.4 Minimum total potential energy approach

In this approach, any structure is in an equilibrium state when the total potential energy of the whole set is minimal. Based on these principles, a constricted gradient algorithm is proposed by Coyette and Guisset[28]. Additionally, Kanno and Ohsaki[29] looked at the structure of a cable net using a minimum complementary energy principle. They did this by treating the stress component as a single variable and taking into account nonlinearities in the geometry and materials. In the same way, Temür et al.[30] used the minimum total potential energy principle to solve the truss problem with geometric nonlinearity, and the Particle Swarm Optimization Algorithm (PSOA) is based on it. The nonlinear solution is provided over MINOS as an optimized code. In recent investigations, Toklu et al.[31] and Branam et al.[32] recently did research that used meta-heuristic algorithms to solve nonlinear problems to minimize total potential energy. The minimum total potential energy approach for nonlinear structural analysis presents a substantial obstacle due to the demands placed on the processing of the technique as well as its complexity. This method intends to reduce the amount of energy that is used by successfully addressing difficult optimization issues, which may be demanding in terms of both time and computing resources for certain structural analysis applications.

Most recent methods of nonlinear structural analysis have limitations regarding computational complexity, applicability to complexity systems, and accuracy in modeling large deformations and internal member forces. The opportunity to circumvent these limits is what makes this study's use of the flexibility technique with Pade approximants remarkable. This approach aims to provide efficient and accurate analysis of nonlinear structural behavior while reducing computational demands and improving applicability to indeterminate systems. To accomplish these goals, we will be conducting comparative studies with current methods, expanding a technique for nonlinear structural analysis that utilizes the flexibility method with Pade approximants, and analyzing both simple and complex cable structures subjected to static loadings. We will also be demonstrating how well this methodology models large deformations and behavior in structural systems.

The outline for this paper is as follows: Section 1 provides a basic overview of the cable net's spatial structure as well as its geometrically nonlinear behavior and response. In addition, a brief overview of the various cable net structure analysis techniques is provided. Section 2 describes the formulation steps for the proposed analysis approach. Section 3 examines numerical examples using the proposed technique, and Section 4 concludes this work.

2. Formulation of the Present Technique

Cable-supported structures are introduced as highly flexible structures that distort significantly when subjected to transverse loads. Consequently, the extra challenge is preferred when analyzing these types of nonlinear geometric structures. The present analysis approach is based on the flexibility method's principles as well as structural mechanics fundamentals. The Pade approximation, one of the best approximations of a rational function of a given order, is used to derive this nonlinear equation. To investigate the potential of rational power series approximations, Frobenius[33] proposed and studied this approximation. Henri Eugene Pade[34] later refined it .The Pade approximation is a standard rational function whose extension is designed to settle as far apart as possible using the primary function's Taylor series expansion. In most cases, the Pade approximation provides a better approximation for the original function and may be useful in situations where the Taylor series does not converge, particularly for functions with poles[35].

Generally, in deriving the analysis methods for systems of cable elements, two element types of cables have been introduced: catenary (continuous and discrete) elements and truss elements. In this formulation, the cable element is considered a general bar (truss element) within the initial prestress t for preventing slack of the member, as shown in Figure 1. Let the bar io- jo with the original length L have the initial end coordinates at (x_{io}, y_{io}, z_{io}) and (x_{jo}, y_{jo}, z_{jo}). After experiencing the deformation, its length becomes L_c in i_c-j_c, and the new end coordinates are (x_{ci}, y_{ci}, z_{ci}) and (x_{cj}, y_{cj}, z_{cj}), as shown in Figure 2. The bar in Figure 1 undergoes deformation after being affected by external loads P_i and P_j at both ends; their horizontal and vertical load components are shown in Figure 2.

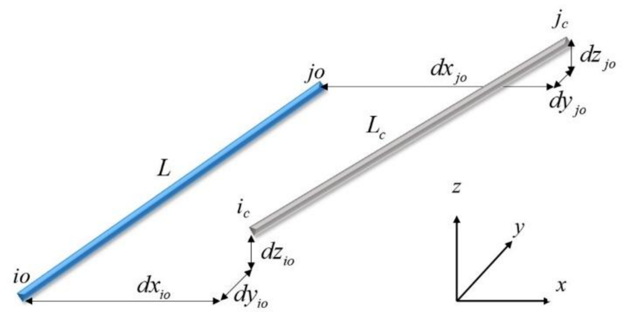

Figure 1: Spatial bar coordinates at original and deformed configuration.

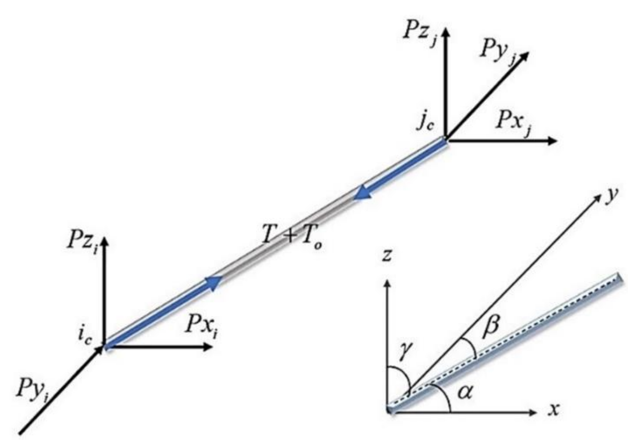

Figure 2: Spatial element equilibrium state at original and deformed configuration.

After loading, the bar experiences bar tension $T$and elongation e over its original length. The abbreviation of the notation is arranged as $(\ )_o = (\ )_{jo} - (\ )_{io}$. Now, by reflecting the new position of the joints, the current length can be written as:

$$L_c = \{(x_o + dx_o)^2 + (y_o + dy_o)^2 + (z_o + dz_o)^2\}^{\frac{1}{2}}$$

$$L_c = (L^2 + 2x_o dx_o + 2y_o dy_o + 2z_o dz_o + dx_o^2 + dy_o^2 + dz_o^2)^{\frac{1}{2}}$$

$$\text{Let } \ H = 2x_o dx_o + 2y_o dy_o + 2z_o dz_o + dx_o^2 + dy_o^2 + dz_o^2$$

$$L_c = (L^2 + H)^{\frac{1}{2}} = L\left(1 + \frac{H}{L^2}\right)^{\frac{1}{2}}$$

The Pade approximation is applied to extend $\left(1+\frac{H}{L^2}\right)^{\frac{1}{2}}$. According to Vazquez-Leal, et al.[35], only the first order of the asymptotic expansion is taken into account because of this method's ability to accelerate or switch from the divergent to the convergent function.

Therefore, the deformed bar length becomes (L_c) becomes:

$$L_c = L\left(\frac{4 + 3\frac{H}{L^2}}{4 + \frac{H}{L^2}}\right)$$

Substituting H, hence: $L_c = L \times \left(\frac{4+\frac{3(2x_o dx_o+2y_o dy_o+2z_o zy_o+dx_o^2+dy_o^2+dz_o^2)}{L^2}}{4+\frac{(2x_o dx_o+2y_o dy_o+2z_o zy_o+dx_o^2+dy_o^2+dz_o^2)}{L^2}}\right)$

The elongation of the bar can be expressed as:

$$e = L_c - L$$

Thus

$$e = L\left\{\left(\frac{4+\frac{3(2x_o dx_o+2y_o dy_o+2z_o zy_o+dx_o^2+dy_o^2+dz_o^2)}{L^2}}{4+\frac{(2x_o dx_o+2y_o dy_o+2z_o zy_o+dx_o^2+dy_o^2+dz_o^2)}{L^2}}\right) - 1\right\} \qquad (1)$$

From the state of equilibrium for the deformed configuration, as shown in Figure 2, the relationship between the internal and external forces and each of their components can be described as shown below:

$$P_i = -(T + t) = -P_j$$

Consequently, for each component in 3D, it becomes:

$$Px_i = -(T + t)\cos\alpha = -Px_j$$
$$Py_i = -(T + t)\cos\beta = -Py_j$$
$$Pz_i = -(T + t)\cos\gamma = -Pz_j$$

Moreover, the terms of $\cos\alpha$,$\cos\beta$ and $\cos\gamma$with neglecting the high order of small displacements, can be in the form:

$$\cos\alpha = \frac{x_o + dx_o}{L_c} = \frac{4x_o L^2 + 4dx_o L^2 + 2x_o^2 dx_o + 2x_o y_o dy_o + 2x_o z_o dz_o}{4L^3 + 6L(x_o dx_o + y_o dy_o + z_o dz_o)}$$

$$\cos\beta = \frac{y_o + dy_o}{L_c} = \frac{4y_o L^2 + 4dy_o L^2 + 2y_o x_o dx_o + 2y_o^2 dy_o + 2y_o z_o dz_o}{4L^3 + 6L(x_o dx_o + y_o dy_o + z_o dz_o)}$$

$$\cos\gamma = \frac{z_o + dz_o}{L_c} = \frac{4z_o L^2 + 4dz_o L^2 + 2z_o x_o dx_o + 2z_o y_o dy_o + 2z_o^2 dz_o}{4L^3 + 6L(x_o dx_o + y_o dy_o + z_o dz_o)}$$

Employing the constitutive relationship between the tensile force of the bar and its elongation can be set up in the form:

$$e = \frac{TL}{EA_o} \qquad (2)$$

where E is the modulus of elasticity, and A_o is the cross-sectional area of the cable.

By equalizing both Equations (1) and (2), the general analytical equation for geometrically nonlinear cable and pin-jointed structures is formulated as below:

$$\left\{\left(\frac{4L^2+3(2x_o dx_o+2y_o dy_o+2z_o dz_o+dx_o^2+dy_o^2+dz_o^2)}{4L^2+(2x_o dx_o+2y_o dy_o+2z_o dz_o+dx_o^2+dy_o^2+dz_o^2)}\right)-1\right\} - \frac{T}{EA_o} = 0 \quad (3)$$

Notably, the proposed method is applicable for the analysis of both simple and complex rigid types of pin-joined spatial structures and can be generalized.

3. Numerical Examples

For validation and presentation of the precision of the proposed nonlinear approach, six numerical examples from the quoted literature have been examined. Then, the results were compared with the findings of the previous analysis techniques. Different tactics may be used to solve the set of nonlinear equations. In the current work, the nonlinear equation's solution was obtained by using MATLAB's fsolve function. By reducing the sum of squares of the components, the function fsolve determines the solution of a simultaneous system of equations. When the sum of squares approaches zero, the set of nonlinear equations is solved.

3.1. Example 1 – Two-linked structure

The two-linked structure is pre-tensioned by 4448.2 N, as shown in Figure 3. Each link has EA_o= 546920 N, which is examined via the present new technique. The middle joint vertical displacement and each internal bar force showed -166.457 mm and 303.193 N, respectively. Accordingly, the analysis of the same structure was presented by Kwan[3] as -166.449 mm and 303.246 N for the same target, respectively. Besides, Levy and Spillers[4] conveyed -166.536 mm and 303.413 N, respectively. The outcomes showed that the current approach has a discrepancy of only 0.004% and 0.04% in displacement with Kwan[3] and Levy and Spillers[4], respectively. At the same time, the tensile force deviations were only 0.02% and 0.07%.

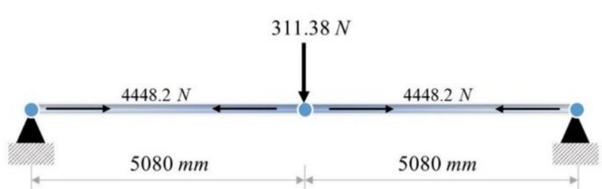

Figure 3: Two-linked structure with nodal external load.

3.2. Example 2 – Flat cable net structure

Figure 4 shows a 3×3 square grid of flat cable net structure, which has been numerically evaluated by numerous studie[3, 16, 31]. It has a 400 mm length of cell sides, an EA of 97970 N, and is prestressed with 200 N. The system has 12 joints. It is supported at its perimeter by 8 joints, leaving 4 inner joints free. It was loaded by 15 N at three positions, as shown in Figure 4. The present formulation is applied to the flat cable net system and then compared with the quoted literature. The results are presented in **Tables 1** and **2** for the joint displacements and cable tensions, respectively, which are very accurate with the other techniques.

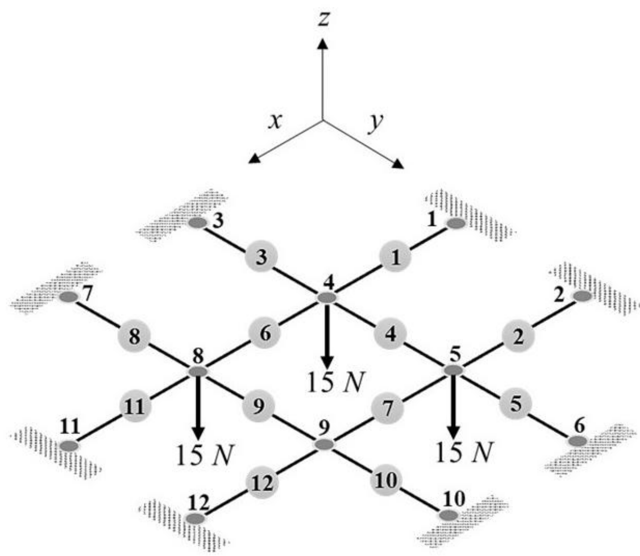

Figure 4: Flat cable net structure with nodal external loads.

Table 1: Nodal displacement (mm) of internal joints for the flat cable net structure in Figure 4.

Node	Present Technique			Kwan [3]			Lewis [16]			Toklu, et al. [31]		
	dx	dy	dz	dx	dy	dz	dx	dy	dz	dx	dy	dz
4	-0.07	-0.07	-12.17	-0.08	-0.08	-12.2	-0.1	-0.1	-12.2	-0.07	-0.07	-12
5	-0.08	0.04	-11.18	-0.08	0.05	-11.2	-0.1	0	-11.2	-0.08	0.04	-11
8	0.04	-0.08	-11.18	0.04	-0.08	-11.2	0	-0.1	-11.2	0.04	-0.08	-11
9	-0.04	-0.04	-5.59	-0.04	-0.04	-5.59	0	0	-5.6	-0.04	-0.04	-5.6

Table 2: Cable tensile forces (N) of the flat cable net structure in Figure 4

Cable	Present technique	Kwan[3]	Lewis[16]
1	227.97	227.97	228.10
2	219.19	219.19	219.30
3	227.97	227.98	-
4	227.94	227.94	228.00
5	228.00	228.01	228.10
6	227.94	227.94	219.20
7	219.14	219.15	219.10
8	219.19	219.19	-
9	219.14	219.15	-
10	219.07	219.08	219.10
11	228.00	228.01	-
12	219.07	219.08	-

3.3. Example 3 – Spatial net structure

In this example, a spatial cable net structure consists of a grid system with 24 m in the x-direction and 16 m in the y-direction, as shown in Figure 5. It has 38 cables with an EA_o of 56×10^6 N and 19.2×10^6 N in the x and y-directions, respectively. Due to its central symmetry, the z-direction coordinates (z-coor.) are given for only a quarter of the net assembly, as presented in **Table 3**. The system is pre-tensioned by 90,000 N in the x- and 30,000 N in the y- direction[3, 16, 31]. The present technique was applied to obtain the displacements after applying the vertical point loads of 6800 N at all internal joints. The attained displacements were compared with the numerical findings by Lewis[16], Abad, et al.[11] and Toklu, et al.[31], as presented in **Table 3**. These results confirmed a remarkable similarity with the established techniques.

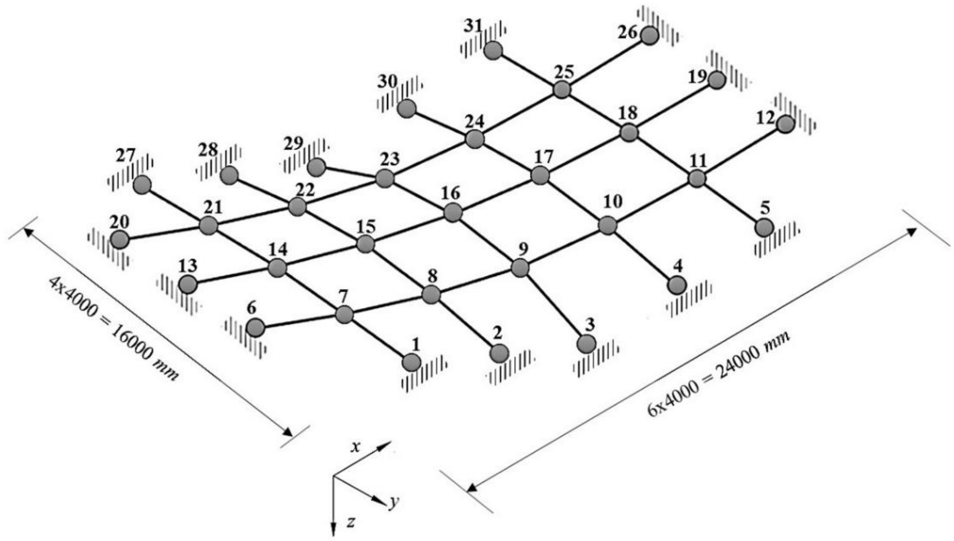

Figure 5: Spatial net structure with nodal labels and panel spacing.

Table 3: Nodal displacements comparison (mm) of the spatial net structure in Figure 5 by different methods

Node	z-coor.	Present Technique			Lewis [16]			Abad, et al. [11]			Toklu, et al. [31]		
		dx	dy	dz	dx	dy	dz	dx	dy	dz	dx	dy	dz
1	1000	---	---	---	---	---	---	---	---	---	---	---	---
2	2000	---	---	---	---	---	---	---	---	---	---	---	---
3	3000	---	---	---	---	---	---	---	---	---	---	---	---
6	0	---	---	---	---	---	---	---	---	---	---	---	---
7	819.5	-5.03	0.40	29.47	-5.14	0.42	30.41	-5.05	0.40	29.6	-5.03	0.40	29.46
8	1409.6	-2.23	0.40	17.12	-2.26	0.47	17.70	-2.23	0.40	17.16	-2.22	0.39	17.18
9	1676.9	0	2.39	-3.19	0	-2.27	-3.62	0	-2.36	-3.19	0	-3.12	-3.19
13	0	---	---	---	---	---	---	---	---	---	---	---	---
14	687.0	-4.93	0	42.88	-4.98	0	43.49	-4.93	0	42.94	-4.92	0	42.84
15	1147.8	-2.55	0	44.32	-2.55	0	44.47	-2.55	0	44.34	-2.55	0	44.27
16	1317.6	0	0	42.14	0	0	41.65	0	0	42.14	0	0	42.08

3.4. Example 4 – Hyperbolic paraboloid net structure

A hyperbolic paraboloid net system featuring 26 joints and 31 cables with 36 degrees of freedom is depicted in Figure 6. A hyperbolic paraboloid net system featuring 26 joints and 31 cables with 36 degrees of freedom is depicted in Figure 6. The axial stiffness of all members is 100200 N. The structure is concentrically loaded by 15.7 N in the z-direction at all internal nodes except 17, 21, and 22. The cable segments carry an amount of 200 N of pretension force. Several authors[3, 5, 14, 16, 31] have numerically and experimentally examined this net system by utilizing different analysis techniques. Such as dynamic relaxation (DR), which is used by Lewis[36] and Kwan[3], while approximation of Taylor series (ATS), elastic catenary cable element in finite element, and total potential optimization were used by Thai and Kim[5] and Toklu, et al.[31], respectively. The results for the vertical displacements of the current and previously published methods are presented in **Table 4**. Comparing the emphasized methodologies to the current results revealed a high level of accuracy and similarity.

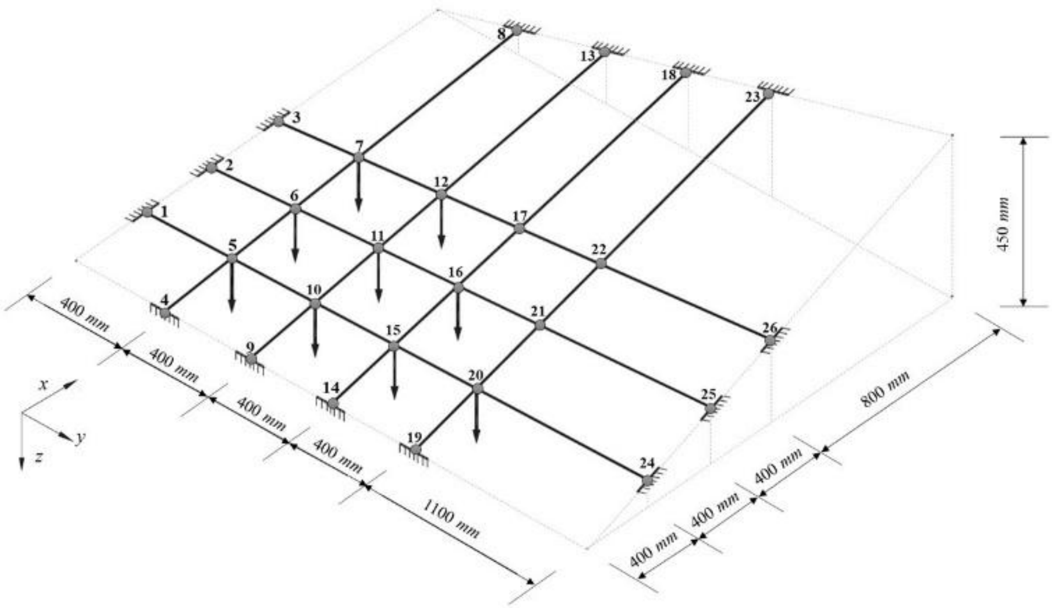

Figure 6: Hyperbolic paraboloid net structure with nodal labels and panel spacing.

Table 4: Nodal displacements comparison (mm) in the z-direction of hyperbolic paraboloid net structure in Figure 6 by different methods.

Node	Present technique	Lewis, et al. [14] Experiment	Lewis, et al. [14] DR	[3] DR	[3] ATS	Thai and Kim [5]	Toklu, et al. [31]
5	19.53	19.50	19.30	19.38	19.52	19.56	19.48
6	24.66	25.30	25.30	25.62	25.35	25.70	25.59
7	23.32	22.80	23.00	22.95	23.31	23.37	23.17
10	25.88	25.40	25.90	25.57	25.86	25.91	25.75
11	34.08	33.60	33.80	33.79	34.05	34.16	33.86
12	29.52	28.80	29.40	29.32	29.49	29.60	29.27
15	25.81	25.20	26.40	25.43	25.79	25.86	25.65
16	31.33	30.60	31.70	31.11	31.31	31.43	30.96
17	21.43	21.00	21.90	21.28	21.42	21.56	21.03
20	21.49	21.00	21.90	21.16	21.48	21.57	21.33
21	20.01	19.80	20.50	19.79	20.00	20.14	19.67
22	14.41	14.20	14.80	14.29	14.4	14.55	14.04

3.5. Example 5 – Saddle net structure

As seen in Figure 7, the preliminary geometry of the saddle net structure consists of 95 joints, 32 of which are constrained at the perimeter, and 142 cables with an EA_o of 44.982×106 N. It has mirror symmetry about both centerlines; each segment has a 5000 mm distance in both x and y-directions, and the z-coordinates (z-coor.) for one-fourth of the structure are given in **Table 5**. The saddle net structure was completed by a tensile prestressing force of 60,000 N and was affected by concentrated loads of 1000 N in x- and y-directions at the half of free nodes (11-15, …., 66-70, and 77-81). The analysis of the proposed method is presented in **Table 5**. After comparing it to previous approaches by[3, 36], Thai and Kim[5], and both discrete and continuous catenary cable models by Abad, et al.[11], it demonstrated good accuracy and was confirmed to be comparable with well-known methods. The maximum percentage of error for the present technique, Kwan[3] and Thai and Kim[5], as compared to experimental work performed by Lewis[36], is not exceeded by 3.87%, while they are 5.81% and 4.91% for the discrete and continuous models of Abad, et al.[11], respectively, as presented in **Table 5**. In most studies, the saddle net is introduced as the most complex cable structure and an outstandingly comparable problem. It is used to confirm the effectiveness of the analysis techniques. As Lewis[16] said, the saddle net analysis failed when using the finite element method because of an ill-condition issue for such a complicated assembly.

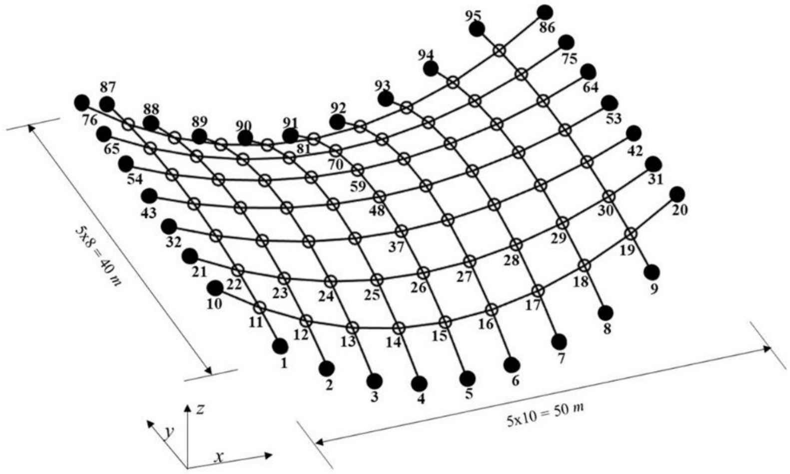

Figure 7: Saddle net structure with nodal labels and panel spacing.

Table 5: Nodal displacements comparison (mm) of saddle net structure in Figure 7 by different methods.

Node	z-coor.	Lewis [16] experiment	Present Technique	Kwan [3]	Thai and Kim [5]	Abad, et al. [11] *d* discrete	Abad, et al. [11] *d* continuous
1	3632	0	0	0	0	0	0
2	2568	0	0	0	0	0	0
3	1808	0	0	0	0	0	0
4	1352	0	0	0	0	0	0
5	1200	0	0	0	0	0	0
10	5000	0	0	0	0	0	0
11	3968	83.53	83.28(0.29)	83.28(0.29)	83.24(0.34)	83.46(0.08)	83.38(0.17)
12	3165	62.85	62.55(0.48)	62.54(0.49)	62.5(0.56)	62.68(0.27)	62.6(0.4)
13	2592	34.57	34.38(0.55)	34.38(0.55)	34.34(0.67)	34.47(0.29)	34.43(0.4)
14	2248	19	18.92(0.42)	18.92(0.42)	18.91(0.47)	19.02(-0.11)	18.96(0.21)
15	2133	12.27	12.21(0.49)	12.22(0.41)	12.21(0.49)	12.29(-0.16)	12.26(0.08)
21	5000	0	0	0	0	0	0
22	4208	98.4	98.27(0.13)	98.27(0.13)	98.23(0.17)	98.57(-0.17)	98.42(-0.02)
23	3592	74.02	73.9(0.16)	73.9(0.16)	73.84(0.24)	74.17(-0.2)	74.03(-0.01)
24	3152	32.84	32.93(-0.27)	32.93(-0.27)	32.89(-0.15)	33.14(-0.91)	33.03(-0.58)
25	2882	11.88	12.15(-2.27)	12.15(-2.27)	12.14(-2.19)	12.33(-3.79)	12.24(-3.03)
26	2800	12.68	12.32(2.84)	12.32(2.84)	12.32(2.84)	12.12(4.42)	12.21(3.71)
32	5000	0	0	0	0	0	0
33	4352	93.19	93.19(0)	93.19(0)	93.15(0.04)	93.56(-0.4)	93.38(-0.2)
34	3848	67.56	67.65(-0.13)	67.65(-0.13)	67.6(-0.06)	68.02(-0.68)	67.84(-0.41)
35	3488	20.81	21.2(-1.87)	21.2(-1.87)	21.16(-1.68)	21.51(-3.36)	21.36(-2.64)
36	3272	15.49	14.89(3.87)	14.89(3.87)	14.89(3.87)	14.59(5.81)	14.73(4.91)
37	3200	36.85	36.09(2.06)	36.09(2.06)	36.07(2.12)	35.77(2.93)	35.9(2.58)
43	5000	0	0	0	0	0	0
44	4400	-	89.36	89.36	89.31	89.75	89.56
45	3933	-	63.44	63.44	63.38	63.84	63.65
46	3600	-	15.16	15.16	15.12	15.49	15.34
47	3400	-	22.99	22.99	22.99	22.65	22.80
48	3333	-	46.11	46.12	46.09	45.74	45.89
52	4400	-	5.93	5.93	5.93	6.34	6.17
72	3152	-	30.37	30.38	30.36	30.07	30.19
81	2133	-	12.21	12.22	12.21	12.29	12.26
85	3968	-	32.67	32.67	32.65	32.89	32.79
85	3968	-	32.67	32.67	32.65	32.89	32.79

()* shows the error percentage of the proposed technique and other quoted techniques concerning Lewi''s experimental work.

3.6. Example 6 – Cantilever truss structure

A simple cantilever truss, as shown in Figure 8, consists of six nodes and ten bars with an axial stiffness of 400,000 N. It is pin-supported at node number one and roller-supported at node number two. The two external point loads are applied on nodes 3 and 5 with quantities of 1000 N and 3000 N in the gravity direction, respectively. The cantilever truss has been previously used by Saeed and Kwan[35] using the linear force method. The linear analysis results were obtained using the least squares solution. The proposed nonlinear force method is applied for analyzing the same cantilever truss, and both of the findings are presented in **Table 6**. SAP2000 software is also used for the purpose of comparison, precision, and validation of the results. The output of the software analysis is basically based on finite element analysis with an improved tangent stiffness matrix, and the results for displacement and member forces are presented in columns 2-4 in **Table 6**. To assess the proposed method's accuracy and utility, the Euclidean Norm index for internal forces (linear and nonlinear) error to internal force from SAP2000 is used. The accuracy evaluation ratio (R_T) of l_2-Norm is found using Equation (5), where T_1 and T_2 are the member forces of the linear force method and SAP2000, respectively. The percent Euclidian norm ratio between linear and SAP2000 was 2.23%, while between the proposed technique and SAP2000, it was 0.05%. These ratios clearly show the precision of the present approach in considering the geometric nonlinearity during the analysis stage of spatial structures. Further, it can be noticed that neglection from the geometric deformability in the linear force method leads to giving the internal force of bar number 7 as zero. That is due to using the equilibrium matrix in its original configuration and the zero coefficient of the state of self-stress found in the null of the equilibrium matrix.

$$R_T = \frac{\|T - T_2\|_2}{\|T_2\|_2} \times 100$$
$$R_{T_1} = \frac{\|T_1 - T_2\|_2}{\|T_2\|_2} \times 100 \qquad (5)$$

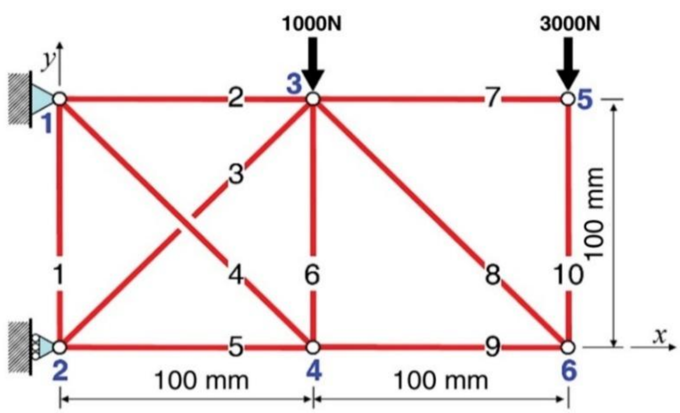

Figure 8: Cantilever truss structure with nodal labels, panels spacing, and nodal loads.

Table 6: Nodal displacements (mm) and internal bar forces (N) computation of the cantilever truss structure in Figure 8 by SAP2000 software, linear and nonlinear force methods

Nodes	Nonlinear Analysis by SAP2000			Linear Analysis by Saeed and Kwan [37]			Nonlinear Analysis present Study			Bars
	Nonlinear Displacement		Member Force	Linear Displacement		Member Force	Nonlinear Displacement		Member Force	
	dx	dy		dx	dy		dx	dy		
2	0	-0.465	1858.4	0	-0.500	2000	0	-0.465	1858.5	1
			5105.6			5000			5107.4	2
3	1.227	-3.149	-2836.8	1.250	-3.164	-2828	1.228	-3.150	-2837.8	3
			2751.8			2828			2752.4	4
4	-1.265	-2.604	-4967.4	-1.250	-2.664	-5000	-1.266	-2.605	-4970	5
			-2055.8			-2000			-2056	6
5	1.061	-9.334	98.3	1.250	-9.286	0	1.061	-9.335	98.683	7
			4304.7			4243			4303.1	8
6	-2.195	-8.532	-3012.1	−2.000	−8.536	-3000	-2.195	-8.533	-3009.4	9
			-2995.5			-3000			-2993.9	10
Euclidian Norm Ratio					R_{T1}	2.23%		R_T	0.05%	

5. Conclusions

A relatively simple numerical analysis method for geometric nonlinearity has been proven in this paper, which provides analysis for cable net and pin-jointed structures with significant accuracy. The method is derived based on the principle of flexibility method via the Pade approximation method, which presents the strength of the technique to lead to vastly accurate results via rational approximate solutions. The proposed technique was applied to five numerical examples: two-lined structure, flat cable net, spatial net, hyperbolic paraboloid net, and saddle net systems. The analysis outcomes of the proposed method were compared to those of several established methods. The proposed approach offers excellent agreement with the other techniques and can be applied to complicated cable net structures. The advantage of the method is its easy access; rather, this approach has been confirmed to be identical to other reputable techniques for the accuracy of the solution and swiftness of the calculating process.

Conflict of interests

None

Authors contribution

Conceptualization, Shna Jabar Abdulkarim, Najmadeen Mohammed Saeed.
Methodology, Shna Jabar Abdulkarim, Najmadeen Mohammed Saeed.
Software, Shna Jabar Abdulkarim.
R.A. validation, Najmadeen Mohammed Saeed.
Formal analysis, Shna Jabar Abdulkarim
Investigation, Shna Jabar Abdulkarim, Najmadeen Mohammed Saeed.
Resources, Shna Jabar Abdulkarim.
R.A. data curation, Shna Jabar Abdulkarim, Najmadeen Mohammed Saeed.
Writing—original draft preparation, Shna Jabar Abdulkarim
R.A. writing—review and editing, Shna Jabar Abdulkarim
Supervision, Najmadeen Mohammed Saeed
R.A.; project administration, Najmadeen Mohammed Saeed

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