International Journal of Bridge Engineering (IJBE), Vol. 11, No. 2, (2023), pp. 53-64

RESPONSE SURFACE MODEL QUANTIFICATION FOR YIELDING STRENGTH IN A BRIDGE GIRDER

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ABSTRACT: The construction of response surface model which represents the behavior of the pre-stressed bridge girder is the objective of this paper. Five parameters which belong to the mechanical properties of both concrete and steel bars in addition to the pre-stressing magnitude would be deployed. The Latin Hypercube sampling method is being utilized to build 25 models for the numerical simulations using ABAQUS program. MATLAB code with the support of least square method are used to determine the response surface model for the yielding strength of the pre-stressed bridge girder. The response surface model will undergo reliability chech by the adoption of coefficient of determination analysis. The results of the regression coefficients manifested a great representation of the behavior of the bridge girder behavior under both static and dynamic loadings. The coefficient of determination for the response surface model was $R^2=1$ which means that the response prediction through the response surface model is totally illustrating the actual behavior of the structural member. The numerical simulations are highly confirming the yielding strain situation in parallel with the results of the response surface model. The results can be further processed to control the design of the pre-stressed bridge girder by the employment of optimization methods.

KEYWORDS: Pre-stressed Concrete; Least Square Method; Latin Hypercube Method; Regression Coefficients; Maximum Principal Strain.

1 INTRODUCTION

Pre-stressed concrete was developed starting in the 1940s in order to solve the problem of tensioned concrete. To increase the strength of the concrete structural element in flexure, pre-stressing steel strands have been combined with conventional steel reinforcement. When it came to bridge construction, the pre-stressed concrete beam was the most often used structural element. Since the turn of the 20th century, a significant number of bridges have been constructed and planned using the pre-stressed concrete approach [1,2]. Up to this point, pre-stressed concrete behaviors have been the subject of multiple studies, and structural modelling has consistently drawn attention. In bonded pre-stressed constructions, tendon strains could be directly obtained from section equilibrium; however, in unbonded pre-stressed concrete, the process was considerably more complex, as tendon strains could only be obtained from the global conditions of the components. Flexural tensile resistance capacity in pre-stressed concrete structures is produced by first compressively stressing the concrete with high-strength steel tendons. By activating the bonding resistance between the two, the released strand's tendency to shorten causes pre-compression in the concrete during precast production. Nonlinear material qualities that reflect the plasticity and deterioration of concrete, as well as the characteristics of the interfacial interface between concrete and steel, should be included in the simulation model. [3,4].

Yapar et al. 2015 [4] modelled a beam using nonlinear finite element analysis, taking into consideration the deterioration and plasticity of the concrete as well as the slip-bond failure behavior of the strands. The model validated the process by accurately and faithfully reproducing the loading history. All of the material and bond models used in this research were derived from experimental data. The simulation results were confirmed using data from real load tests. The response of the damaged beam after local bonded composite patch repair was also considered, in addition to the behaviour of the beam up to the limit state. To do this, pre-stressed concrete beam specimens were fabricated in a laboratory and assessed both before and after they were fixed with bonded composite patches. It was noticed that the test findings of the virgin beam and forecasts made using finite elements agree effectively.

Pham and Hong, 2022 [3] used nonlinear finite element models to investigate strain evolutions with consideration to concrete plasticity and concrete-damaged plasticity in bonded and unbonded pre-stressed reinforced concrete beams. Non-convergence problems were commonly encountered in pre-stressed beam investigations because of the material's tendency to soften at large deformations and the high increase in contact stresses throughout the stress transferring phases. The investigation yielded information that can enhance the ductility and material efficiency of pre-stressed beam designs. The surface-to-surface contact between pre-stressing steels and concrete is thought to accurately replicate the pre-stressing effects, with the least degree of slippage. Padmarajaiah and Ramaswamy, 2002 [5] applied non-linear finite elemental analysis for 3D model to estimate the flexural responses of partially and fully pre-stressed high-strength concrete beams. Kim et al. 2008 [6] examined the flexural response of pre-stressed concrete beams that had carbon fiber-reinforced polymer (CFRP) sheets strengthening them. They are using non-linear finite element analysis to investigate the beams' ductility and cracking behavior. Badawy et al. 2020 [7] investigated three responses for pre-stressed concrete beams: failure scenarios, maximum load capacity, and beam deflection at critical locations. Bonopera et al. 2019 [8] developed and modelled a high strength, parabola-shaped pre-stressed concrete beam for a bridge with an unbonded tendon. To determine the stability of the fundamental frequency, the pre-stressed concrete beam was subjected to free vibration and various pre-stressing pressures. Noble et al. 2016 [9] conducted a dynamic impact test using various pre-stressing force scenarios and nine specimens of post-tensioned concrete beams for the static three-point flexure test, and presented the results. For additional investigation and analysis, the behavior of the bridge girder under static and dynamic loads against yielding is vital and critical. The response surface model is being used in this research work to evaluate and forecast the pre-stressed bridge girder's response. In addition to the pre-stressing parameter, five more parameters for the mechanical characteristics of steel and concrete would be taken into account. Using the ABAQUS program, finite element models of the pre-stressed concrete girder would be created. The entailing models would be ready for the estimation procedure using the Latin hypercube sampling approach. To determine the maximum main strain for the plastic yielding behavior of the pre-stressed bridge girder, MATLAB algorithms and the least squares method are being utilized

2 RESPONSE SURFACE MODEL

A response surface model is made up of several statistical and mathematical techniques that are necessary to enhance and optimize a function's system output. The response surface model depends on the computational model of the system being studied when taking numerical simulations into account [10, 11]. The relationship between the system output (y) and the number of involving parameters (x1, x2,..., xk) is authorized by the response surface model. Usually, the relationship in question remains unclear until a multi-polynomial equation of the following type is constructed:

$$\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})\boldsymbol{\alpha} + \epsilon \tag{1}$$

where f (x) is a vector function of k elements made up of powers and cross-products of powers of x1, x2,..., xk, and x = (x1, x2,..., xk). α is a vector of k unknown coefficients, and ϵ is a random error with a mean value of zero. As a result, the mean system response is authorised by the value f(x)α. There will be two terms in the response surface model: a linear term and a quadratic one. There are two different kinds of terms in the quadrilateral term: primary terms and interaction terms. The following would be the representation for the linear term:

$$\boldsymbol{y} = \boldsymbol{\alpha}_{\text{o}} + \sum_{i=1}^{k} \boldsymbol{\alpha}_i x_i + \epsilon \tag{2}$$

And the quadratic term is expressed by:

$$\boldsymbol{y} = \boldsymbol{\alpha}_0 + \sum_{i=1}^{k} \boldsymbol{\alpha}_i \boldsymbol{x}_i + \sum_{i<j} \sum \boldsymbol{\alpha}_{ij} \boldsymbol{x}_i \boldsymbol{x}_j + \sum_{i=1}^{k} \boldsymbol{\alpha}_{ii} \boldsymbol{x}^2{}_i + \boldsymbol{\epsilon} \tag{3}$$

It is necessary to conduct a set of n trials for a predetermined number of factors in order to obtain the value of y for each experiment, which represents the response surface model y, in order to develop the response surface model. The following matrix is made for the experiments of order (n × k), and it is used with the least squares approach to find $\boldsymbol{\alpha}$ vector that contains all the response surface model coefficients:

$$\boldsymbol{\alpha} = (\boldsymbol{X'X})^{-1}\, \boldsymbol{X'y} \tag{4}$$

where $\boldsymbol{X'}$ is the transpose matrix, and the matrix $\boldsymbol{X}$ contains the set values of the generated models for the involving parameters.

3 LATIN HYPERCUBE METHOD

When we have *p* experiments for *m* parameters, the Latin Hypercube is authorized by a matrix with *p* x *m* dimension. Each column is a modification of *p* levels which are equally spaced. The *p* levels are considered in such away to be − (*p* − 1)/2, − (*p* −3)/2,...,(*p* −3)/2,(p − 1)/2. If we have an *p* × *m* matrix, then the Latin hypercube will be: *L* = (l_{ij}), a Latin hypercube design *D* = (d_{ij}) for the design space $[0, 1)^m$ is generated through:

$$d_{ij} = \frac{l_{ij} + \frac{(n-1)}{2} + u_{ij}}{n} \tag{5}$$

where $= 1, \ldots., n, j = 1, \ldots, m,$ and u_{ij} 's are random numbers which are independent from [0, 1). The popularity of Latin hypercube designs was mostly attained due to the theoretical proof for the reduction of the variance during numerical integration [13].

4 FINITE ELEMENT MODEL

The finite element model of the pre-stressed bridge girder a cross-section (30.48*15.24) cm and 8.74 m length (see Figure 1). The girder is being loaded at four points. The concrete girder has two tendons for pre-stressing purpose with 1.12 cm diameter and four steel bars, two of them positioned at the top region and the rest two steel bars are positioned in the bottom region. Each of the steel bars has 2 cm diameter. The boundary conditions of the pre-stressed girder is constrained in three directions at one support and constrained in two directions at the other support to accommodate the generated stresses due to external loadings.

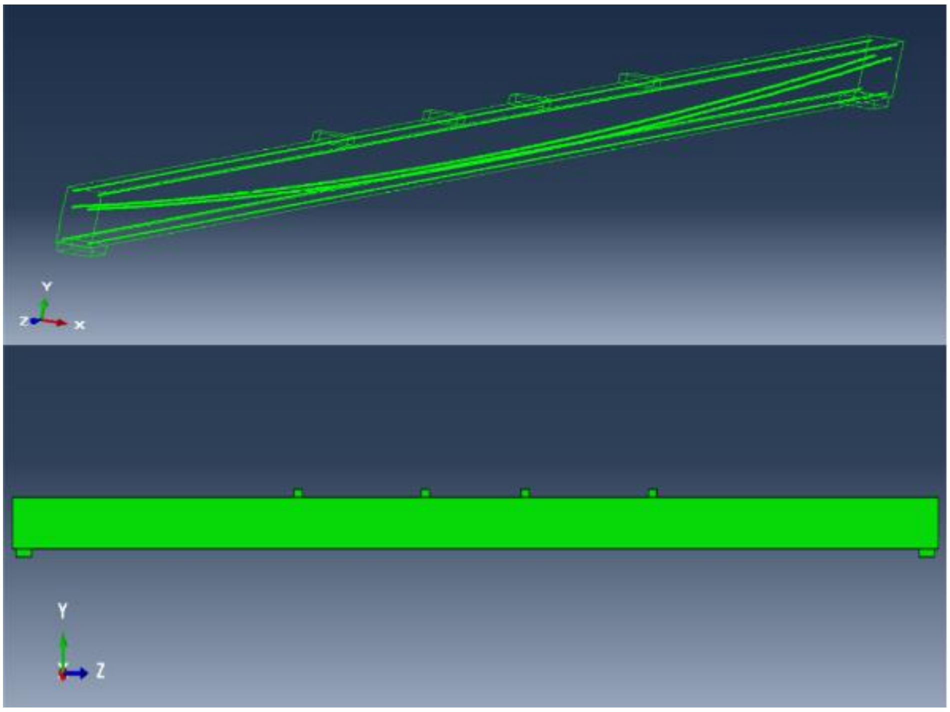

Figure 1. The finite element model of the bridge girder

4.1 Material mechanical properties

For the response surface model, the mechanical characteristics of the materials would be produced as a range for every parameter, as indicated in Table 1. Five variables are involved, all of which are controlled, and the analysis process is completely certain. Consequently, a specific tool for regulating the pre-stressed bridge girder's yielding strength will be developed by the response surface model. The Latin Hypercube method and the least squares approach would be used to organize the 25 models for the numerical simulation in ABAQUS. The result would be the highest primary strain that might cause the pre-stressed concrete bridge girder's concrete to yield plastically.

Table 1. Ranges of parameters

Parameter	**Symbol**	**Range Values**
Concrete Density (ρ_c) kg/m^3	$\mathbf{X_1}$	2200 - 2600
Concrete Young's Modulus (E_c) GPa	$\mathbf{X_2}$	24 - 35
Steel Density (ρ_s) kg/m^3	$\mathbf{X_3}$	7800 - 8000
Steel Young's Modulus (E_s) GPa	$\mathbf{X_4}$	190 - 230
Pre-stressing (MPa)	$\mathbf{X_5}$	1.2 - 2

4.2 Latin hypercube models

The following Table 2 contains the arrangement of the 25 models generated for the involving parameters with their value ranges which has been clearly stated in Table 1. The 25 models would be assigned in ABAQUS program to run the analysis and collect the output of maximum principal strain for them in order to be utilize for further processing in the response surface model stage.

Table 2. Latin hypercube models arrangement

	X_1	X_2	X_3	X_4	X_5
Model 1	2483.33	34.08	7816.67	201.67	1.37
Model 2	2233.33	27.21	7833.33	216.67	1.27
Model 3	2566.67	29.50	7941.67	213.33	1.43
Model 4	2583.33	32.71	7900.00	230.00	1.67
Model 5	2333.33	24.92	7866.67	225.00	1.80
Model 6	2466.67	33.63	7891.67	208.33	1.73
Model 7	2300.00	25.38	8000.00	228.33	1.60
Model 8	2200.00	32.25	7908.33	206.67	1.50
Model 9	2283.33	33.17	7958.33	211.67	1.97
Model 10	2416.67	24.46	7916.67	203.33	1.57
Model 11	2250.00	35.00	7858.33	205.00	1.83
Model 12	2600.00	30.88	7966.67	218.33	1.93
Model 13	2550.00	28.13	7850.00	226.67	1.87
Model 14	2216.67	24.00	7875.00	198.33	1.53
Model 15	2450.00	34.54	7991.67	223.33	1.77
Model 16	2516.67	29.04	7825.00	193.33	1.90
Model 17	2383.33	26.75	7933.33	195.00	2.00
Model 18	2500.00	27.67	7883.33	190.00	1.40
Model 19	2366.67	31.79	7925.00	220.00	1.33
Model 20	2533.33	25.83	7808.33	210.00	1.47
Model 21	2316.67	29.96	7950.00	200.00	1.23
Model 22	2350.00	31.33	7841.67	221.67	1.63
Model 23	2433.33	30.42	7983.33	196.67	1.70
Model 24	2400.00	26.29	7975.00	215.00	1.20
Model 25	2266.67	28.58	7800.00	191.67	1.30

5 RESULTS AND DISCUSSION

The sections that follow would look at data analysis related to the maximum principal strain of the pre-stressed concrete bridge girder and the construction of the response surface model, as well as the calculation of the coefficient of determination of the response surface model's reliability.

5.1 Maximum principal strain

The greatest primary strain output for each model has been gathered, which is the minimum value utilized to develop the response surface model for the yielding strength of the bridge girder (see Table 3 and Figure 2). The minimum value of the maximum primary strain was 0.049388 in model 24, while the maximum value was 0.066802 in model 21. As a result, when the design necessitates an optimization procedure, we should consider the minimal value in model 24 for the control of concrete yielding damage.

Table 3. Maximum principal strain

Model	Maximum Principal Strain ( )
Model 1	0.064947
Model 2	0.050292
Model 3	0.065188
Model 4	0.050479
Model 5	0.058109
Model 6	0.055528
Model 7	0.058113
Model 8	0.055531
Model 9	0.058003
Model 10	0.055427
Model 11	0.058218
Model 12	0.055632
Model 13	0.065065
Model 14	0.065070
Model 15	0.050384
Model 16	0.050387
Model 17	0.056683
Model 18	0.056687
Model 19	0.056893
Model 20	0.056897
Model 21	0.066802
Model 22	0.063422
Model 23	0.051426
Model 24	0.049388
Model 25	0.056790

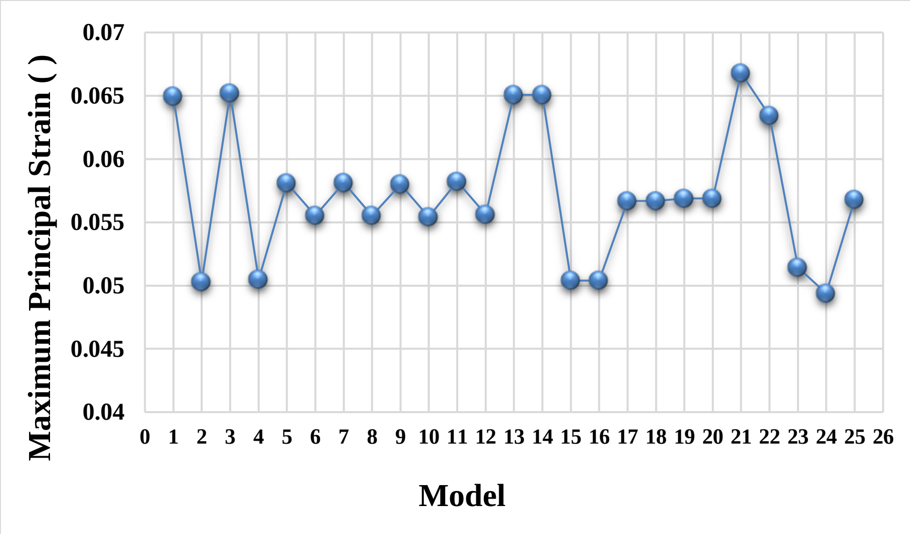

Figure 2. Maximum principal strain - 25 models

5.2 Response surface model results

The obtained data for the maximum principal strain response surface model for the pre-stressed concrete bridge girder consists mostly of the regression coefficients for each term of the main equation (see Table 4).

The resulting response surface model is used to forecast the yielding strength of the concrete and then manage the structural system's behavior under static and dynamic loadings (see Equation 6).

Table 4. Regression coefficients

Response Surface Model Coefficient	Maximum Principal Strain
α_0	-3.05404856803161717238026540144346654415130615234375
α_1	0.0022548228164532015636856154827682985342107713222503662109375
α_2	0.08522438810321512903112051162679563276469707489013671875
α_3	-0.00033600198674454810100764579061660697334446012973785400390625
α_4	-0.0012102963804176696156755799194115752470679581165313720703125
α_5	0.8938852393136034191201133580761961638927459716796875
α_{11}	0.0000000708634367063396206449447500136384903157704684417694807052612
α_{22}	-0.0004090320699980461334466230738371450570411980152130126953125
α_{33}	0.00000000935306903074600485529268819109849353310437436448410153388977
α_{44}	-0.0000251873925824143897087784710331348492218239698559045791625977
α_{55}	0.00892827617960814494402210783619011635892093181610107421875

α_{12}	0.00000577741961169782050963946926080971877581760054454207420349121
α_{13}	-0.000000344404948094588687900212322506865447735435736831277608871 46
α_{14}	0.000000542039733976360994278801200274564564551837975159287452697754
α_{15}	-0.00011615585148323984815942055970339197301655076444149017333 9844
α_{23}	-0.0000104269469845710325921726846498671648078016005456447601318359
α_{24}	0.0000642022440619942808036693659978766390850069001317024230957031
α_{25}	-0.00394329572467769576271656717381119960919022560119628906 25
α_{34}	0.000000728323893383036702677284148638170435674510372336953878402 71
α_{35}	-0.000111387607258076285084678891390552735174424014985561370849609
α_{45}	0.00172977481073343731035607540036380669334903359413146972656 25

$$\boldsymbol{Response\ Surface\ Model} = \alpha_0 + \alpha_1 * X1 + \alpha_2 * X2 + \alpha_3 * X3 + \alpha_4 * X4 + \alpha_5 * X5 + \alpha_{11} * X1^2 + \alpha_{22} * X2^2 + \alpha_{33} * X3^2 + \alpha_{44} * X4^2 + \alpha_{55} * X5^2 + \alpha_{12} * X1 * X2 + \alpha_{13} * X1 * X3 + \alpha_{14} * X1 * X4 + \alpha_{15} * X1 * X5 + \alpha_{23} * X2 * X3 + \alpha_{24} * X2 * X4 + \alpha_{25} * X2 * X5 + \alpha_{34} * X3 * X4 + \alpha_{35} * X3 * X5 + \alpha_{45} * X4 * X5 \quad (\boldsymbol{6})$$

The above response surface model is utilized to predict the maximum principal strain in the pre-stressed concrete girder. This model would be checked for efficiency then it is used to predict the maximum principal strain output. By determining the coefficient of determination for the response surface model which is named by the symbol R^2, the reliability of the response surface model is obtained.

5.3 Coefficient of determination

The response surface model calculation process for the maximum principal strain response for the bridge girder requires determining the coefficient of determination R2 by comparing the magnitudes of the maximum principal strain obtained in the numerical simulation with the magnitudes of the maximum principal strain determined by the response surface model. R2 = 1 was the coefficient of determination of the maximum primary strain in the bridge girder (see Figure 3). It is obvious that the response surface model precisely replicates the real behaviour of the pre-stressed bridge girder 100%.

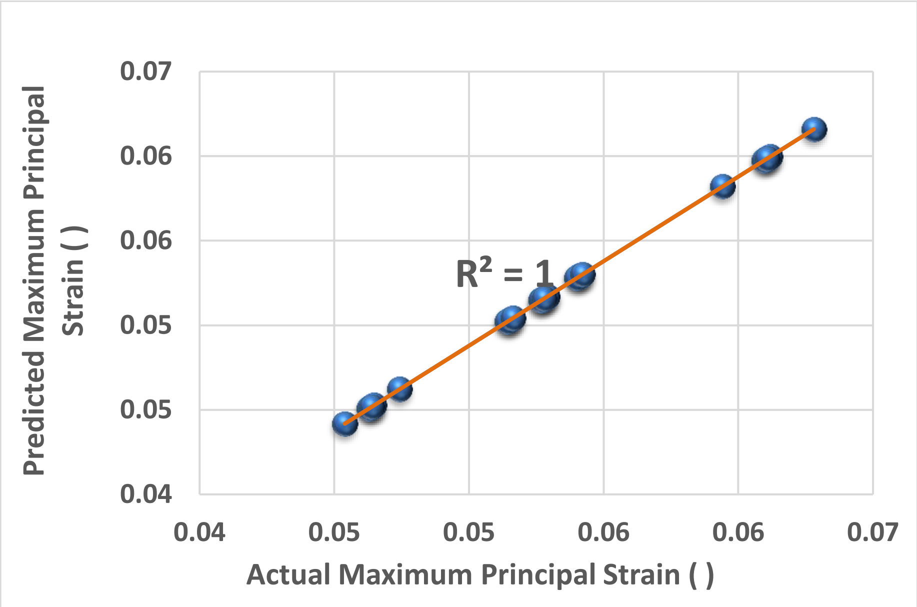

Figure 3. Coefficient of determination-maximum principal strain

6 NUMERICAL SIMULATION RESULTS

6.1 Maximum principal strain

The models chosen for the numerical simulations would be the least, middle, and maximum scenarios for the greatest principal strain output. We can clearly see the difference in yielding in each example. The largest primary strain in the minimal case is 0.049388, which occurred in model 24. The strain is plainly visible in the bottom of the bridge girder (see Figure 4). When evaluating the middle scenario, which is in model 25, the strain increases vertically and horizontally, reaching 0.05679 (see Figure 5). Finally, the maximum scenario happens in model 21, and the maximum major strain is more expanding horizontally towards the top face of the bridge girder and horizontally as well to reach 0.066802 (see Figure 6). The data can then be used to investigate the control of the yielding of the pre-stressed concrete bridge girder using optimization methods to create the optimized model for the structural member's design.

Figure 4. Minimum case- maximum principal strain

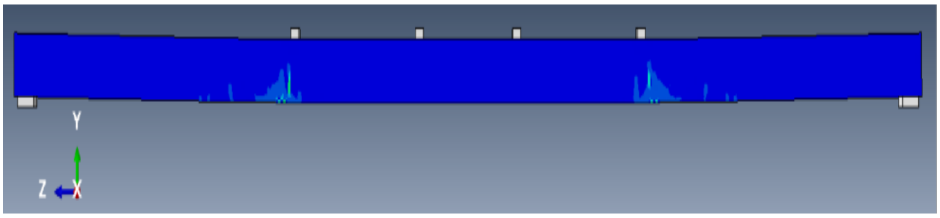

Figure 5. Middle case- maximum principal strain

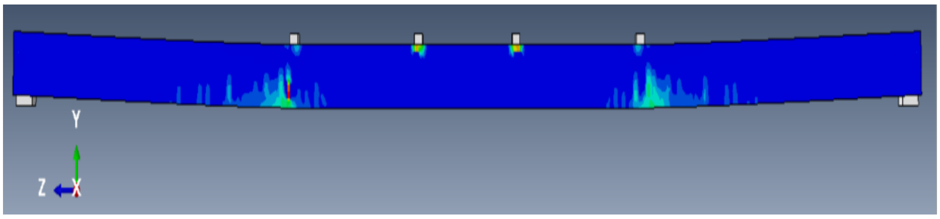

Figure 6. Maximum case- maximum principal strain

7 CONCLUSIONS

Based on the findings and analysis, the following conclusions have been reached:

1- The Latin Hypercube sampling method produced an efficient arrangement for the pre-stressed bridge girder models, resulting in the identification of a critical tool for the response surface model.

2- By combining the efforts of both the numerical simulation results and the least square method, a robust tool for predicting the behavior of the structural system under static and dynamic loads was created.

3- The response surface model for the yielding strength of the bridge girder was calculated efficiently and validated by the coefficient of determination with an efficiency of 99%, indicating the highest reliability.

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