Eurasian J. Sci. Eng., 2024, 10(1), 23-34
https://doi.org/10.23918/eajse.v10i1p3

Published by Tishk International University
Available at: https://eajse.tiu.edu.iq/submit/

RESEARCH ARTICLE Open Access

Nonlinear Prestress of Space Cable Net Structures

Shna Jabar Abdulkarim[1,2]*, and Najmadeen Mohammed Saeed [2,3]

[1] Civil Engineering Department, Erbil Technical Engineering College, Erbil Polytechnic University, Erbil, Iraq.
[2] Civil Engineering Department, University of Raparin, Rania, Iraq.
[3] Civil Engineering Department, Faculty of Engineering, Tishk International University, Erbil, Iraq.

Article History
Received: 25.11.2022
Revised: 22.12.2022
Accepted: 18.02.2024
Published: 28.03.2024
Communicated by: Asst. Prof. Dr. Abubakar M. Ashir
**Email address:*
shna.jabar@uor.edu.krd
**Corresponding Author*

Abstract:
Cable-net structures are used for many structural purposes, such as stadiums, roofs, bridges…etc. They are lightweight structures that can be used in unique construction at an effective cost. Geometrical nonlinearity governs the performance of cable net systems. This particular system can equilibrate applied loads by undergoing significant deformations with small strains. Therefore, the cable-net structures require to attain a suitable degree of prestressing to prevent cables from slacking and to obtain specific geometry and function. The effective numerical approach is applied for computing the desired level of prestress for a three-dimensional cable-net model and a conical cable-net model. The targeted prestress is achieved considering the nonlinear behavior of cables. The nonlinear member variation is introduced as a second-order function of displaced joints. Then used in determining the desired prestress. Two numerical examples are conducted using the present technique and the nonlinear analysis of SAP2000. Both of the analysis outcomes for the models showed a very well agreement with reaching the target. However, using the Euclidean norm index with a value of 0.0809 in the first example confirmed that the current technique is more approachable to the desired prestress. In addition, when the value of actuation is pre-determined and used in computing the degree of prestress, both the present approach and SAP2000 software work equivalently, as seemed in the second example, which showed 0.04% of the maximum difference in the prestress computation.

Keywords: *Cable-net; Prestress; Geometric Nonlinearity; Self-Equilibrate; Force Method.*

1. Introduction

Cable-net structures are tensile structures whose stability is dependent on the axial tensile force alone. The majority of cable-net structures are kinematically indeterminates; therefore, they rely on their geometrical flexibility follow-on their self-equilibrated state [1]. Prestressing is essential in cable-net systems to attain the looked-for shape, stiffness, and stability [2]. Moreover, the level of prestressing has a direct influence on indicating the load tolerance capability, expenses, and shape deformation [3]. The cable-net form is generally determined via its nodal positions, and it has highly flexible geometry. Thus, taking into account the geometric nonlinearity during the prestressing is crucial [4-7].

Many research publications have been conducted to find out the optimal degree of prestressing for various tensile structures. Pellegrino [8] developed an approach based on the linear force method, assuming small deformation, to find out the prestressed mechanism and the loading conditions that have an effect on causing cable looseness. Kwan and Pellegrino [9] used the least squares analysis technique to calculate the state of prestress in particular and optimal situations with preselected actuators equal to the number of self-stress states. However, You [10] prestressed cable structure by altering the member length of the cables, and similarly controlled the nodal positions by charging the prestress degree above the lower limit. Dong and Yuan [11] used a space grid system to determine the preliminary internal force during the pre-tensioning process. Subsequently, they computed the controlled prestress force via pretensioning the elements of the space grid that were requested in the design.

Consequently, when large deformation and geometric nonlinearity are engaged, the linear techniques become inaccurate or unacceptable [2, 6, 12-15]. Hence, a more validated technique is desired to deal with the large deformability and geometric nonlinearity issues. Utilizing a Simulated Annealing Algorithm (SAA) with the developed nonlinear force method. An iteration procedure is applied to attain the nonlinear controlling of the nodal of the prestressed cable-net [6]. In a later study, a Dynamic Relaxation Method (DRM) was inserted as a linear force method for displacement control to reach nonlinear nodal controlling for geometrically nonlinear prestressed systems [2]. Guo and Zhou [12] took benefit from the nonlinear behavior of a negative Gaussian-curvature cable dome to propose a pretension algorithm relying on the iterative procedure amid the targeted internal stress and required prestressing level.

The cited researche demonstrates that in order to determine the prestressing level of statically and kinematically indeterminate cable structures, an algorithm, and iteration from developed linear approaches are validated and applied. It was suggested that, despite mechanical and modeling imprecision, the quantity of member alteration created could result in self-equilibrated conditions [9, 16]. Therefore, this work is carried out to apply an effective nonlinear technique derived based on the force method that both the equilibrium and compatibility matrices are formulated depending on the topology at a fully deformed shape. The targeted prestressing level is computed by the nonlinear amount of actuation in 3D and conical cable-net models. The calculated amount of total actuation was then validated by comparing it with the nonlinear analysis results of SAP2000.

The layout of this article is as follows. Section 1 presents a general introduction and review of the literature. In Section 2, the basic equations of the force method are introduced and related to nonlinear member actuation. The short form of a nonlinear single element of member actuation for structures with nonlinear geometrical behavior is given in Section 3.Two numerical examples of the space and conical cable-net models using the nonlinear technique and SAP2000 are presented in Section 4. Finally, some conclusions are presented in Section 5.

2. Nonlinear Force Method

The general self-equilibrated prestressing state and the compatibility equations can be expressed as:

(1) $$\boldsymbol{H}(\boldsymbol{d})\boldsymbol{T}_o = 0$$

(2) $$\boldsymbol{G}(\boldsymbol{d})\boldsymbol{d} = \boldsymbol{e}_o(\boldsymbol{d})$$

where $\boldsymbol{H}(\boldsymbol{d})$ is the equilibrium matrix at the deformed configuration, $\boldsymbol{T}_o$ is the internal force. $\boldsymbol{G}(\boldsymbol{d})$ is the compatibility matrix at the deformed configuration, and $\boldsymbol{d}$ and $\boldsymbol{e}_o(\boldsymbol{d})$ are nonlinear nodal displacement and member actuation, respectively. It is proved by the virtual work that $\boldsymbol{G}^T(\boldsymbol{d}) = \boldsymbol{H}(\boldsymbol{d})$ [17].

3. Nonlinear Member Actuation

We now proceed to derive the nonlinear actuation equations for a general bar in 3D space, as shown in Figure 1. The cable element is determined as a bar that contains tensile axial force while prestressed. Deem a bar *o-h* with node coordinates (x_o, y_o, z_o) and (x_h, y_h, z_h), and initial length l_o facing deformability during prestressing by shortening amount of e_o so that it ends up in new position *o'-h'* with end coordinates ($x_{o'}$, $y_{o'}$, $z_{o'}$) and ($x_{h'}$, $y_{h'}$, $z_{h'}$) and new length of l', see Figure 1. Adopting the abbreviation $(\ )_{ho} = (\ )_h - (\ )_o$, the member length after prestress is:

(3) $$l' = l_o - e_o$$

Similarly, looking back to Figure 1, the l' can also be expressed as:

(4) $$l' = \sqrt{\left(x_{ho} - dx_{ho}\right)^2 + \left(y_{ho} - dy_{ho}\right)^2 + \left(z_{ho} - dz_{ho}\right)^2}$$

Equation (4) can be rewritten as $l' = \left(l_o^2\left(1 + B/l_o^2\right)\right)^{\frac{1}{2}}$, where $B = \left(dx_{ho}^2 + dy_{ho}^2 + dz_{ho}^2 - 2x_{ho}dx_{ho} - 2y_{ho}dy_{ho} - 2z_{ho}dz_{ho}\right)$.

Utilizing Taylor's series in expanding B gives an infinite-term summation of the function by deriving the term of that function at a specific point. This means that the original function will be equal to the sum of its derived terms at this point [13].

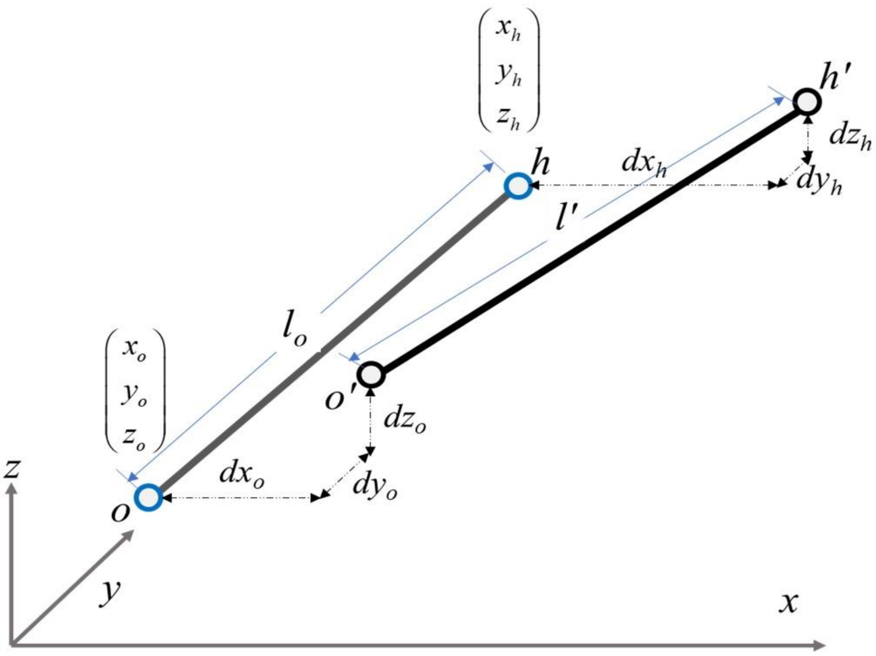


Figure 1: Displacement and member length during prestressing process

Now applying the Taylor series expansion on l' yields [7, 13, 14] $l' = l_o\left(1 + 1/2\left(B/l_o^2\right) - 1/8\left(B/l_o^2\right)^2 + \ldots\right)$, then substituting the B into the expansion and equalizing (3) by (4) gives the nonlinear required amount of member actuation as a second order function to the nodal position in the deformed configuration as below:

(5) $$e_o = \frac{x_{ho}dx_{ho} + y_{ho}dy_{ho} + z_{ho}dz_{ho}}{l_o} - \frac{dx_{ho}^2 + dy_{ho}^2 + dz_{ho}^2}{2l_o} + \frac{{x_{ho}}^2 dx_{ho}^2 + y_{ho}^2 dy_{ho}^2 + {z_{ho}}^2 dz_{ho}^2}{2l_o^3} + \frac{x_{ho}y_{ho}dx_{ho}dy_{ho}}{l_o^3} + \frac{x_{ho}z_{ho}dx_{ho}dz_{ho}}{l_o^3} + \frac{y_{ho}z_{ho}dy_{ho}dz_{ho}}{l_o^3}$$

In (5), some terms with trivial effects have been ignored. Referring to (1), the member internal axial force in equilibrium due to large deformation and member length alteration after prestressing process can be set to the x-, y-, and z-components as:

$$\begin{aligned} T_o x &= T_o \cos\theta \\ T_o y &= T_o \cos\gamma \\ T_o z &= T_o \cos\varphi \end{aligned} \tag{6}$$

where $\cos\theta = x_{ho} - dx_{ho}/l_o - e_o$, $\cos\gamma = y_{ho} - dy_{ho}/l_o - e_o$, and $\cos\varphi = z_{ho} - dz_{ho}/l_o - e_o$. The relation between the nonlinear amount of member actuation and the member prestress can be presented in the constitutive relationship as:

$$\boldsymbol{FT_o} = \boldsymbol{e_o}(\boldsymbol{d}) \tag{7}$$

where $\boldsymbol{F}$ is the flexibility matrix and can be provided from the element modulus of elasticity $\boldsymbol{E}$, cross-sectional area $\boldsymbol{a_o}$, and initial member length $\boldsymbol{l_o}$ as $l_o/\boldsymbol{Ea_o}$ [6, 18-20]. The general form of the algebraic system can be written as:

$$\begin{aligned} \boldsymbol{FT_o} = {} & \frac{x_{ho}dx_{ho} + y_{ho}dy_{ho} + z_{ho}dz_{ho}}{l_o} - \\ & \frac{dx_{ho}^2 + dy_{ho}^2 + dz_{ho}^2}{2l_o} + \frac{x_{ho}{}^2 dx_{ho}^2 + y_{ho}^2 dy_{ho}^2 + z_{ho}{}^2 dz_{ho}^2}{2l_o^3} + \\ & \frac{x_{ho}y_{ho}dx_{ho}dy_{ho}}{l_o^3} + \frac{x_{ho}z_{ho}dx_{ho}dz_{ho}}{l_o^3} + \frac{y_{ho}z_{ho}dy_{ho}dz_{ho}}{l_o^3} \end{aligned} \tag{8}$$

Equation (8) is a system of nonlinear equations that require mathematical nonlinear solver methods such as substitution or elimination techniques. Since, the arrangement of the nonlinearity is defined as power 2, any technique that solves a nonlinear algebraic system can be used; however, in this work, fsolve in MATLAB is used.

4. Numerical Examples

The validation of the technique is examined through two numerical examples, namely 3D cable-net and conical cable-net models. The same two models were analyzed in SAP2000 software using geometrical nonlinear analysis regarding large deformation. The comparison has been made between the outcomes of the proposed technique and SAP2000 and tabulated in the following subsections.

4.1 3D Cable-net Model

The 3D cable-net model shown in Figure 2, has 14 nodes that the x, y, and z coordinates are shown in Table 1. Eight of them are pinned and shown with black solid connectors. It consists of 21 members with the axial stiffness of 40,000 *N*. The targeted degree of prestress (T_o) for all the cables is determined as shown in the 6th column in Table 1. The prestressing process is performed by pre-indicating the required member pretension force for all members. The present technique has been applied to attain the targeted prestress (T_o^*) using fsolve in MATLAB, and the results came out as shown in the 7th column in Table 1. The other findings of the present technique, such as the 3D nodal displacements of the free nodes and the required amount of nonlinear member actuation, are tabulated in Table 2. As a practical consideration, only members 7, 8, 9, 10, 19, 20, and 21 are selected as actuators, while the remaining cables are set to be unchangeable in length. The total amount of shortening is 13.1514 mm. Later, the results are validated by nonlinear analysis results of the same model using SAP2000 finite element analysis software. At this stage, the same quantity of members shortening is assigned to the same cables. The outcomes of the computed prestress and nodal displacements are presented in the 8th column in Table 1 and columns 2-7 in Table 2.

For testing the accuracy and closeness to the desired prestress level, the Euclidean norm (l_2-norm) [15] is used as an evaluation index, as presented in Table 1. The l_2-norm is defined as $\|T_o - T_o^*\|_2$ and $\|T_o - T_o'\|_2$ to indicate the discrepancy between the vectors of targeted prestress and the calculated prestress by the present technique and SAP2000 respectively. The smaller value of the Euclidean norm is 0.0809 with the present technique showing that it is more accurate and more approachable to the desired target in comparison to T_o'. Regarding the nodal displacements, the computed dx, dy, and dz by the present technique have a very good agreement in comparison to the SAP2000 results as shown in Table 2. Figure 3 shows the amount of discrepancy between the targeted prestress and the present technique prestress, as well as $\boldsymbol{T_o}$ and SAP2000 prestress for the cables labeled in column 1 of Table 1. For some cables (10 members) the computed prestress by SAP2000 is closer to the targeted $\boldsymbol{T_o}$, while for the other 11 members, the computed one by the present technique is closer, as confirmed through the Euclidean norm index.

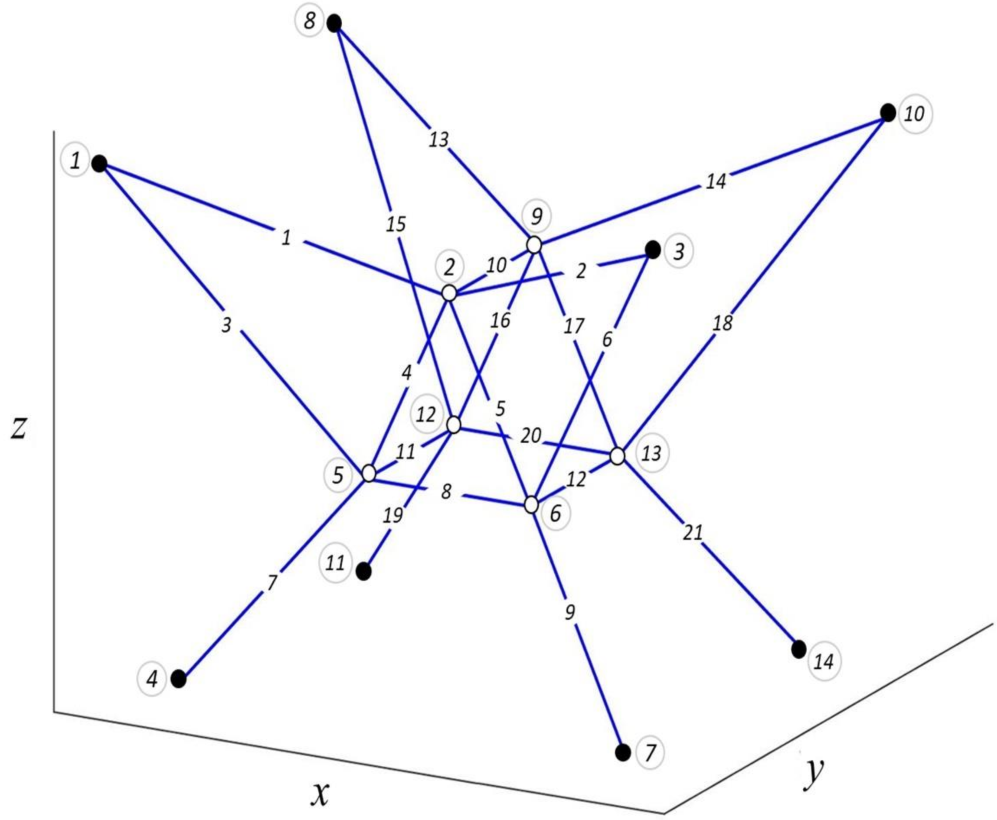


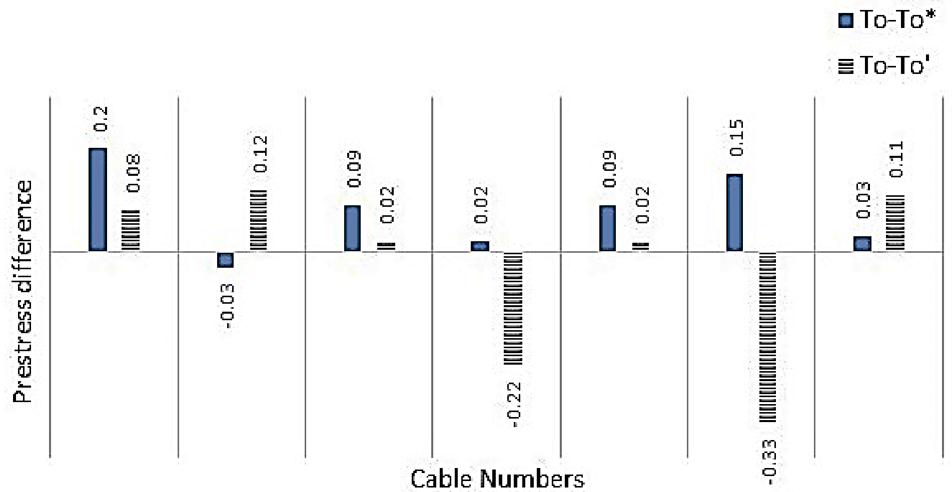
Figure 2: 3D cable-net model with labelled nodes and members

Table 1: Nodal coordinates, cable prestress and l_2 - norm

Nodes	Coordinates			Cables	Prestress (N)		
	x (mm)	y (mm)	z (mm)		T_o	T_o* *Present Technique*	T_o' *SAP2000*
1	0	720	720	1,2,13,14	51	50.8	50.92
2	900	270	540				
3	1800	720	720	3,6,15,18	29	29.03	28.88
4	180	540	0				
5	630	270	270	4,5,16,17	12.7	12.61	12.68
6	1170	270	270				
7	1620	540	0	7,9,19,21	51.1	51.08	51.32
8	0	-720	720				
9	900	-270	540	8,20	50.5	50.41	50.48
10	1800	-720	720				
11	180	-540	0	10	44.5	44.35	44.83
12	630	-270	270				
13	1170	-270	270	11,12	38.1	38.07	37.99
14	1620	-540	0				
$\lVert T_o - T_o^* \rVert_2$					0.0809		
$\lVert T_o - T_o' \rVert_2$					0.1909		

Table 2: Nodal displacement and cable actuation

Nodes	Displacement (mm)						Actuator	e_o (mm)
	Present Technique			SAP2000				
	dx	dy	dz	dx	dy	dz	7	-1.3796
2	0	-2.5476	-1.0199	0	-2.5422	-1.0220	8	-0.9696
5	0.1446	0.2569	-1.3495	0.1433	0.2559	-1.3368	9	-1.3796
6	-0.1446	0.2569	-1.3495	-0.1433	0.2559	-1.3368	10	-5.6938
9	0	2.5476	-1.0199	0	2.5422	-1.0220	19	-1.3796
12	0.1446	-0.2569	-1.3495	0.1433	-0.2559	-1.3368	20	-0.9696
13	-0.1446	-0.2569	-1.3495	-0.1433	-0.2559	-1.3368	21	-1.3796
Total Actuation (mm)								13.1514



1	3	4	7	8	10	11
2	6	5	9	20		12
13	15	16	19			
14	18	17	21			

Figure 3: Prestress difference between targeted prestress and present technique and also targeted prestress and SAP2000 for 3D cable-net model

4.2 Conical Cable-net Model

As a second example, a conical cable-net structure as shown in Figure 4 prestressed by the present approach. The system consists of 24 cables labeled as shown in figure 5, and they have an axial stiffness (*EA*) of 10^4 N/mm^2. The model has 18 nodes, as shown in Figure 4, and the coordinates are tabulated in columns 2-4 of

Table 3: Conical cable-net nodal coordinates and displacements. Joints 2, 4, 6, and 13-18 are restrained against x-, y-, and z- directions translation, while joints 1, 3, and 5 are restrained only in vertical (z) direction. Nodes 6-12 are free to move in all directions. The prestress of this model started by determining the member shortening of cables 19-24 by 9 mm and cables 7-12 by 5 mm, as shown in column 4 of Table 4, which results in 84 mm of the total amount of member actuation that prevents any slack of the cables. Via using equation (8), the required pretension for all members is attained and presented in Table 4 (column 2). The exterior nodal displacement also came out, as presented in columns 5-7 of

Table 3: Conical cable-net nodal coordinates and displacements.

Similarly, the conical cable-net system is modeled and prestressed in SAP2000 software with the same given properties of joints, connectivity members, and supports. The identical amount of member actuation as member deformation in the load case is assigned to the above-mentioned cables. Then it is analyzed by selecting nonlinear geometrical considerations with large deformability. The joint displacement output is shown in

Table 3: Conical cable-net nodal coordinates and displacements (columns 8-10), and the member pretensions output is presented in Table 4(column 3). The maximum difference percent of the displacement resultant between the present technique and SAP2000 is 0.01%, while the maximum discrepancy percent for prestress level is 0.04% as shown in figure 6. These findings show the great consistency and precision of the proposed approach in computing the required degree of prestress without causing any slack in any member.

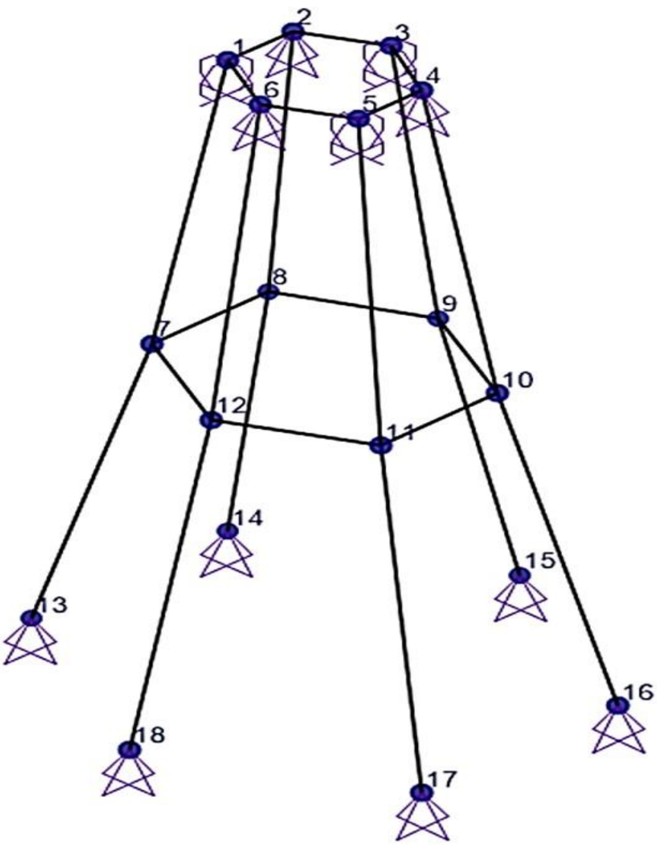


Figure 4: Conical cable-net model with labelled nodes

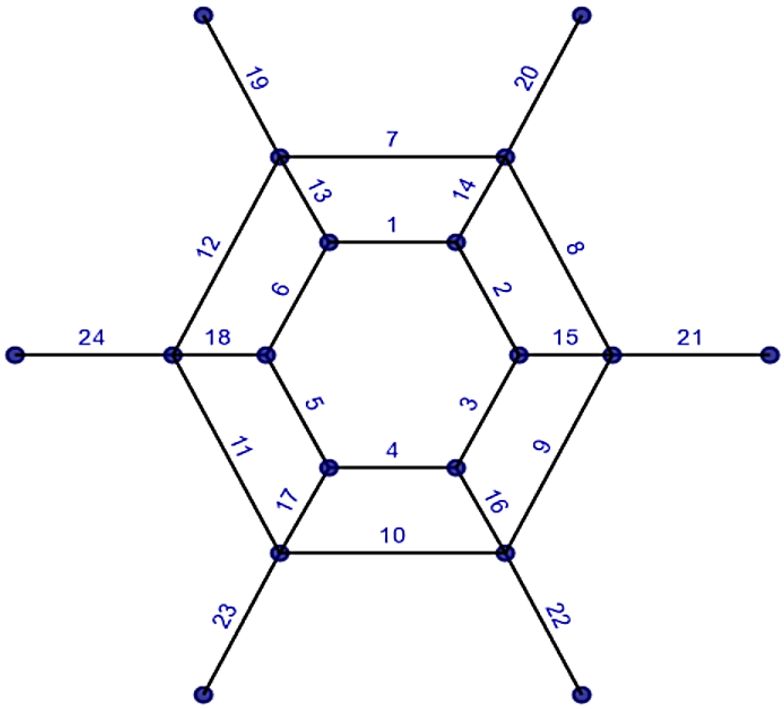


Figure 5: Top view of the conical cable-net model with labeled cables

Table 3: Conical cable-net nodal coordinates and displacements

Nodes	Coordinates mm			Displacements (mm)					
				Present technique			SAP2000		
	x	*y*	*z*	*dx*	*dy*	*dz*	*dx*	*dy*	*dz*
1	500	800	2000	-0.13605	0.25208	0	-0.13605	0.25207	0
2	700	800	2000	0	0	0	0	0	0
3	800	600	2000	0.36521	0	0	0.36520	0	0
4	700	400	2000	0	0	0	0	0	0
5	500	400	2000	-0.13605	-0.25208	0	-0.13605	-0.25207	0
6	400	600	2000	0	0	0	0	0	0
7	420	950	1000	2.42820	-4.61710	-5.68460	2.42818	-4.61715	-5.68466
8	780	950	1000	-2.40600	-4.57380	-5.64970	-2.40595	-4.57384	-5.64983
9	950	600	1000	-3.92790	0	-5.30870	-3.92774	0	-5.30872
10	780	250	1000	-2.40600	4.57380	-5.64970	-2.40596	4.57384	-5.64983
11	420	250	1000	2.42820	4.61710	-5.68460	2.42818	4.61714	-5.68466
12	250	600	1000	3.86180	0	-5.26860	3.86166	0	-5.26859
13	300	1200	0	0	0	0	0	0	0
14	900	1200	0	0	0	0	0	0	0
15	1200	600	0	0	0	0	0	0	0
16	900	0	0	0	0	0	0	0	0
17	300	0	0	0	0	0	0	0	0
18	0	600	0	0	0	0	0	0	0

Table 4: Conical cable-net elements, member prestress, and element actuation

Cables	Prestress (N)		eo (mm)
	Present technique	SAP2000	
1,4	6.8104	6.8103	0
2, 3	7.3148	7.3147	
5,6	7.3676	7.3674	
7,10	4.6076	4.6075	-5
8,9	5.6949	5.6948	
11,12	5.6917	5.6917	
13,17	46.3540	46.3542	0
14,16	46.5490	46.5492	
15	45.7450	45.7446	
18	45.9650	45.9646	
19,23	47.5640	47.5639	-9
20,22	47.7590	47.7586	
21	46.7130	46.713	
24	46.9320	46.9316	
Total actuation (mm)			84

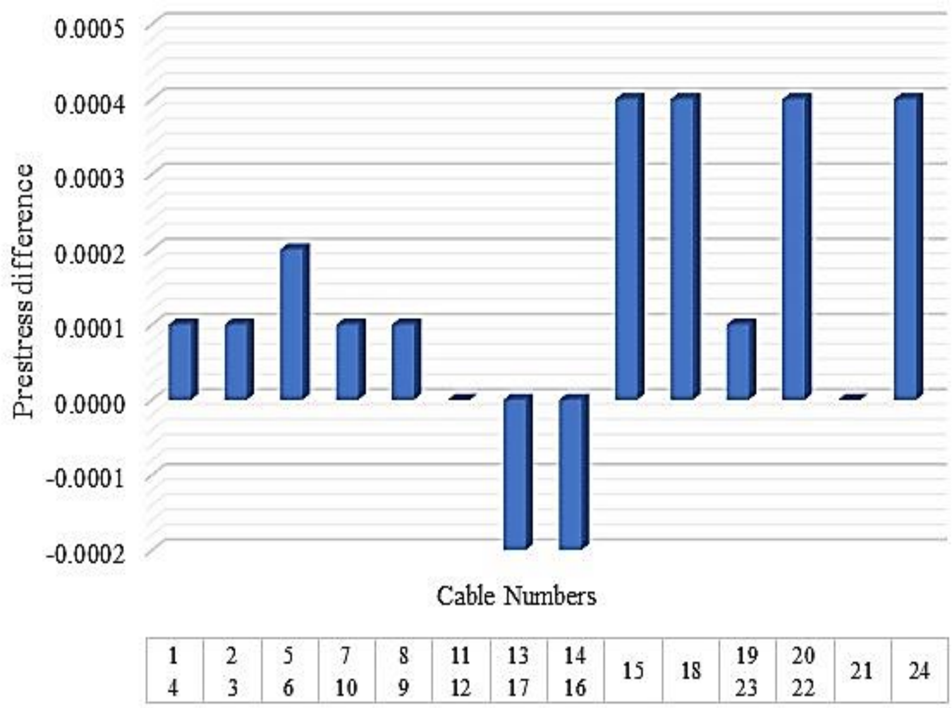


Figure 6: Prestress difference between the present technique and SAP2000 for the conical cable-net model

5. Conclusion

In this paper, the application of the nonlinear prestressing technique has been performed on a three-dimensional cable-net system and a conical-shaped cable-net system. The present approach is based on the nonlinear force method, using nonlinear member actuation as a function of deformed nodal displacement. The findings are validated with the outcomes of nonlinear geometric finite element analysis by SAP2000. The applied technique can be used either by indicating the pre-level of desired prestress as in the 1st example or by introducing the amount of member actuation as in the 2nd example. The Euclidean norm index is employed to specify the precision of the technique. It got out to be 0.0809 and 0.1909 when the closeness of the present technique and SAP2000 was measured concerning the targeted prestress, respectively. Generally, the conclusions can be summarized as:

1. The present method is applicable for prestressing space cable structures.
2. The applied approach is very accurate in computing the targeted prestress when it is pre-determined. It can be seen in the 2nd cable-net numerical example that the maximum difference between the targeted and computed current prestress was 0.2, while it was 0.33 with the computed prestress via SAP2000.
3. The present technique is equivalent when the amount of member actuation is pre-indicated. It is confirmed in the conical cable-net example that the maximum discrepancy between the applied technique and SAP2000 was only 0.0004.
4. The slack of the cables can be prevented through the required member actuation. The total actuation for attaining a particular state of prestressing for the 3D cable-net was 13.1514 mm using 7 cables out of 21 members, but it was 84 mm for the conical cable-net via 12 members shortening.
5. These results are not the optimal solution for reaching the desired prestress level. Although, trying different cables may result in a more optimal set of actuators.

6. Conflict of Interest

The authors have no conflicts of interest.

References

[1] Hanaor A. Prestressed pin-jointed structures—flexibility analysis and prestress design. Computers & structures 1988;28:757-69. https://doi.org/10.1016/0045-7949(88)90416-6.

[2] Li A, Liang X, Yuan X. Force Method for Displacement Adjustment of Prestressed Cable Structures with Dynamic Relaxation Method. DEStech Transactions on Engineering and Technology Research 2017. https://doi.org/10.12783/dtetr/iceta2016/7136.

[3] Cinquini C, Contro R. Prestressing design method for cable net structures. Engineering Structures 1985;7:183-9. https://doi.org/10.1016/0141-0296(85)90045-8.

[4] Saeed N, Manguri A, Szczepanski M, Jankowski R. Non-Linear Analysis of Structures Utilizing Load-Discretization of Stiffness Matrix Method with Coordinate Update. Applied Sciences 2022;12:2394. https://doi.org/10.3390/app12052394.

[5] Saeed NM. Displacement Control of Nonlinear Pin-Jointed Assemblies Based on Force Method and Optimization. AIAA Journal 2022;60:1024-31. https://doi.org/10.2514/1.J060568.

[6] Xu X, Luo Y. Non-linear displacement control of prestressed cable structures. Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering 2009;223:1001-7. https://doi.org/10.1243/09544100JAERO455.

[7] Kwan ASK. A new approach to geometric nonlinearity of cable structures. Computers & Structures 1998;67:243-52. https://doi.org/10.1016/S0045-7949(98)00052-2.

[8] Pellegrino S. Analysis of prestressed mechanisms. International Journal of Solids and Structures 1990;26:1329-50. https://doi.org/10.1016/0020-7683(90)90082-7.

[9] Kwan A, Pellegrino S. Prestressing a space structure. AIAA journal 1993;31:1961-3. https://doi.org/10.2514/3.11876.

[10] You Z. Displacement control of prestressed structures. Computer Methods in Applied Mechanics and Engineering 1997;144:51-9. https://doi.org/10.1016/S0045-7825(96)01164-4.

[11] Dong S, Yuan X. Pretension process analysis of prestressed space grid structures. Journal of Constructional Steel Research 2007;63:406-11. https://doi.org/10.1016/j.jcsr.2006.04.006.

[12] Guo J, Zhou D. Pretension simulation and experiment of a negative Gaussian curvature cable dome. Engineering Structures 2016;127:737-47. https://doi.org/10.1016/j.engstruct.2016.09.002.

[13] Levy R, Spillers WR. Analysis of geometrically nonlinear structures: Springer Science & Business Media; 2003. https://doi.org/10.1007/978-94-017-0243-0.

[14] Abdulkarim SJ, Saeed NM. Nonlinear technique of prestressing spatial structures. Mechanics Research Communications 2023;127:104040. https://doi.org/10.1016/j.mechrescom.2022.104040.

[15] Xue Y, Wang Y, Xu X, Wan H-P, Luo Y, Shen Y. Comparison of different sensitivity matrices relating element elongations to structural response of pin-jointed structures. Mechanics Research Communications 2021;118:103789. https://doi.org/10.1016/j.mechrescom.2021.103789.

[16] Fraddosio A, Pavone G, Piccioni MD. A novel method for determining the feasible integral self-stress states for tensegrity structures. Curved and Layered Structures 2021;8:70-88. https://doi.org/10.1515/cls-2021-0007.

[17] Calladine CR. Buckminster Fuller's “Tensegrity” structures and Clerk Maxwell's rules for the construction of stiff frames. International Journal of Solids and Structures 1978;14:161-72. https://doi.org/10.1016/0020-7683(78)90052-5.

[18] Luo Y, Lu J. Geometrically non-linear force method for assemblies with infinitesimal mechanisms. Computers & Structures 2006;84:2194-9. https://doi.org/10.1016/j.compstruc.2006.08.063.

[19] Yuan X, Liang X, Li A. Shape and force control of prestressed cable-strut structures based on nonlinear force method. Advances in Structural Engineering 2016;19:1917-26. https://doi.org/10.1177/1369433216652411.

[20] Saeed NM, Kwan ASK. Simultaneous displacement and internal force prescription in shape control of pin-jointed assemblies. AIAA journal 2016;54:2499-506. https://doi.org/10.2514/1.J054811.