Effect of Steel Reinforcement on the Minimum Depth-Span Ratio

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Abstract: Building codes and standards include methods and provisions for deflection control and determining minimum thickness of slabs and beams, also determine the immediate and long or time-dependent deflection. The ACI code depth-span limitations tabulated for normal weight concrete and specified yield strength of steel. For other values of steel yield strength and lightweight concrete correction factors are provided. In this study, correction factors are suggested to include the effect of tensile reinforcement on the depth-span ratio in addition to the ACI code correction factors.

Keywords: Deflection, Depth-Span Ratio, Deflection Control

1. Introduction

Deflection control of building codes and standards include methods and provisions for calculating deflection as minimum thickness of slabs and beams. There are two approaches to deflection control.

i. Indirect method, by assuming suitable upper limits on the depth-span ratio which is satisfactory for many cases of spans, loads, load distribution, member size and properties.
ii. Direct method by calculating the deflection for the actual case and compare the results with the specific limitations that permitted by the codes and standards.

Generally, the deflection is occurring during the normal service life of the member due to full dead load and some fraction of live load. The ACI code (ACI Committee, American Concrete Institute, & International Organization for Standardization, 2014) and other design codes and specifications calculate the deflection under loads up to the full-service load to ensure that stresses in the stream fiber in both steel and concrete remain within the elastic ranges, i.e. the un-cracked section properties are used in the calculations of the immediate deflection. Then the long term or time dependent deflection is calculated due to concrete creep and shrinkage along the life of the structure.

ACI code (ACI Committee, American Concrete Institute, & International Organization for Standardization, 2014) provides the minimum depth for one-way slabs and beams as shown in Table 1 for non-prestressed condition, normal weight concrete with density ($w_c = 145$ pcf or $2320\ kg/m^3$) and steel reinforcement yield strength ($f_y = 60000$ psi or 414 MPa). Correction factors are used for light weight concrete with density in the range $(90-115)$ pcf or $(1440 - 1840)$ kg/m^3 and yield strength other than (60000 psi):

$$\lambda_w = 1.65 - 0.005\, w_c \geq 1.09 \quad (1)$$

$$\lambda_y = 0.4 + \frac{f_y}{100000} \quad (2)$$

Table 1: Minimum thickness (h) ACI code limitation [1-10]

Support type	One-way slab	Beam
Simply supported	$L/20$	$L/16$
One end continuous (Propped)	$L/24$	$L/18.5$
Two ends continuous (Fixed ended)	$L/28$	$L/21$
Cantilever	$L/10$	$L/8$

The minimum thickness calculated by the code provisions to ensure that the beam or slab will be stiff enough and the deflection will be within the permissible range. Generally, deflections are influenced by load, span, beam cross section properties, material properties and support conditions (simply supported, fixed or free). Elastic deflection can be expressed in the following general form (Nilson, Darwin & Dolan, 2010; Wight, 2016; McCormac & Brown, 2015):

$$\Delta = \frac{f\left(\text{load, span, support conditions}\right)}{EI} \quad (3)$$

Where:

$EI =$ the flexural rigidity of the member ($N.mm^2$).
$E =$ modulus of elasticity of the material (MPa).
$I =$ moment of inertia of the cross section (mm^4).

$f\left(\text{load, span, support conditions}\right)$ is a function of the load, span and support conditions, which is determined by elastic analysis, Table 2 shows the maximum deflection of different type beams and loadings (Hibbeler & Kiang, 2015; Spiegel & Limbrunner, 2003; Ghali, Neville & Brown, 2003). Factors affecting of reinforced concrete beams and slabs are loadings, material property E, section property I, boundary conditions or support conditions and time dependent factors due to creep and shrinkage on concrete, also the deflection can be controlled by addition of the steel reinforcement bars in tension and compression zones or using pre-stressing concrete. Lee et al. (2013) compared provisions of different codes and standards about minimum thickness, they concluded that the CSA and ACI provisions have limited application and the proposed equation is recommended for calculation the minimum thickness. Beal (1983) presented an approximated depth-span ratio for the preliminary design specifications in term of (M/bd^2) rather than (A_s/bd) to include the effect of steel design stress. Shehata, Shehata and Garcia (2003) presented a theoretical study for the minimum steel ratio that are required for bending, shear and torsion for beams with different concrete strengths. Ho, Kwan and Pam (2004) developed a simplified method for providing minimum flexural ductility and evaluation of maximum values of tension steel ratio and neutral axis depth corresponding to the proposed minimum curvature ductility factor for various concrete grades and steel yield strengths. Akmaluddin (2011) presented an improvement model of the effective moment of inertial to predict the short-term deflection of reinforced light weight concrete beam. The proposed model is verified and compared with experimental results of nine beams, good agreement is obtained with the experimental results and in some cases, have similar trend to the ACI and SNI provisions.

Table 2: Maximum deflection of different types of loads and beams $= k\,w L^4 / EI$

Beam type	Loading	*k*
Simply supported	Uniform distributed load	5/384
One end continuous (Propped)	Uniform distributed load	1/185
Two ends continuous (Fixed ended)	Uniform distributed load	1/384
Cantilever	Uniform distributed load	1/8
Simply supported	Concentrated load at mid-span	1/48
One end continuous (Propped)	Concentrated load at mid-span	1/192
Two ends continuous (Fixed ended)	Concentrated load at mid-span	1/48√5
Cantilever	Concentrated load at tip	1/3

Un-cracked section property (I_{ut}) is used in the calculation of deflection up to cracking moment when the tensile stress at the extreme fiber reached to the tensile strength of the concrete (f_r), but beyond this limit, effective moment of inertia (I_e) is used which is lied between cracking and un-cracked moment of inertia, as given in the following equation:

$$I_e = \left(M_{cr}/M_a\right)^3 I_{ut} + \left[1-\left(M_{cr}/M_a\right)^3\right] I_{cr} \tag{4}$$

where: $I_{cr} =$ cracked transformed section moment of inertia (mm^4).

$I_{ut} =$ un-cracked transformed section moment of inertia (mm^4).

$I_e =$ effective moment of inertia (mm^4).

$M_a =$ maximum bending moment due to the service load ($kN.mm$).

$M_{cr} =$ cracking bending moment due to the service load ($kN.mm$) and equal to:

$$M_{cr} = \frac{f_r\, I_{ut}}{y_t} \tag{5}$$

$f_r =$ modulus of rupture of the concrete (MPa).

$y_t =$ distance from the neutral axis of the section to the extreme fiber at the tension face (mm).

2. Methodology

In this study a modification factor is suggested on the depth-span ratio which is recommended in ACI code including the effect of the tension reinforcement on the determination of the moment of inertia taking f and E in equation (3) are constant. For rectangular section without reinforcement: $I_{ut} = bh^3/12$ while for beams with tension reinforcement:

$$y' = \frac{bh\left(\frac{h}{2}\right) + (n-1)A_s d}{bh + (n-1)A_s}$$

$$I_{ut} = bh^3/12 + bh\left(y' - h/2\right)^2 + (n-1)A_s\left(d - y'\right)^2$$

where: $b =$ width of the beam (mm).
$h =$ total depth of the beam (mm).
$d =$ effective depth of the beam (mm).
$A_s =$ area of the tension reinforcement (mm^2).
$y' =$ distance from the extreme fiber compression to the neutral axis of a concrete beam.
$n = E_s/E_c$
$E_s =$ Modulus of elasticity of steel $= 200000$ MPa.
$E_c =$ Modulus of elasticity of concrete (MPa) $= 4730\sqrt{f_c'}$
$f_c' =$ cylinder compressive strength of the concrete (MPa).

By equating the moment of inertia of the beam with tension reinforcement with the equivalent section without reinforcement which gives the same deflection, the new equivalent depth (h_1) is determined form the following equation:

$$bh_1^3/12 + bh_1\left[\frac{bh_1\left(\frac{h_1}{2}\right) + (n-1)(\rho bd_1)d_1}{bh_1 + (n-1)(\rho bd_1)} - h_1/2\right]^2 + (n-1)(\rho bd_1)\left[d_1 - \frac{bh_1\left(\frac{h_1}{2}\right) + (n-1)(\rho bd_1)d_1}{bh_1 + (n-1)(\rho bd_1)}\right]^2 = bh^3/12 \quad (6)$$

where: $d_1 = h_1 -$ central cover
$\rho =$ reinforcement index ratio $= A_s/bd$

Assuming ($d_1 = 0.85h_1$), equation (6) leads to polynomial equation of 5th degree which is solved using Newton – Raphson method to find the value of the equivalent depth (h_1).

3. Numerical Example

Take a beam with the following dimensions:

$b = 10\,\text{in}\ (250\ \text{mm})$, $h = 20\,\text{in}\ (500\ \text{mm})$, $f_c' = 4000\ \text{psi}\ (28\,\text{MPa})$ and $f_y = 60000\,\text{psi (414 Pa)}$. Taking reinforcement $\rho = \rho_b$ (balance reinforcement index).

$$\rho_b = \left(\frac{0.85\beta_1 f_c'}{f_y}\right) \times \left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right) \quad (7)$$

Substituting eq. (7) in eq. (6), the following equation is obtained:

$$h_1^5 + 11.77h_1^4 + 28.42h_1^3 - 8000h_1^2 - 54264h_1 - 92108.4 = 0 \quad (8)$$

this equation is solved to determine $h_1 = 18.69\,\text{in}\,(474.7\,\text{mm})$.

By applying the same procedures, the equivalent depth (h_1) is determined for different value of reinforcement ratio indices (ρ) as shown in Table 3.

Table 3: Results of Calculations of the numerical example

ρ	ρ/ρ_b	h	h_1	$\alpha = h_1/h$	% Reduction
0	0.00	20 in (500 mm)	20 in (500 mm)	1.000	0.00
$0.5\rho_b$	0.50	20 in (500 mm)	19.32 (490.73)	0.966	3.40
ρ_t	0.63	20 in (500 mm)	19.2 (487.68)	0.960	4.00
ρ_{max}	0.72	20 in (500 mm)	19.09 (484.89)	0.955	4.55
ρ_b	1.00	20 in (500 mm)	18.69 (474.73)	0.935	6.55

$$\rho_{max} = \left(\frac{0.85\beta_1 f_c'}{f_y}\right) \times \left(\frac{\varepsilon_u}{\varepsilon_u + 0.004}\right) \quad (9)$$

$$\rho_t = \left(\frac{0.85\beta_1 f_c'}{f_y}\right) \times \left(\frac{\varepsilon_u}{\varepsilon_u + 0.005}\right) \quad (10)$$

Figure 1 shows the relation between the ratio (h_1/h) and the reinforcement indices ratio (ρ/ρ_b), as shown the ratio (h_1/h) decreased while the reinforcement ratio (ρ/ρ_b) is increased which means that smaller depth is required as the tension reinforcement area is increased. The best fit equation obtained from figure (1) is:

$$\left(h_1/h\right)=\left(1-0.065\,\rho/\rho_b\right) \quad (11)$$

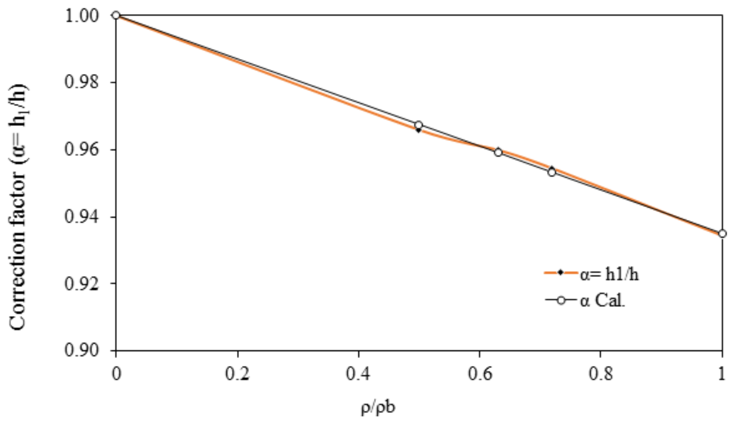

Figure 1: Effect of reinforced indices ratio on the correction factor (α)

Equation (11) represent the correction factor for the depth (h) which is determined form ACI code limitation Table 1. The above equation (11) can be written in another way:

$$h_1 = \alpha\, h_{ACI} \quad (12)$$

where (α) is the correction factor for the steel reinforcement effect as shown in Table 4.

$$\alpha=\left(1-0.065\,\rho/\rho_b\right) \quad (13)$$

or

$$h_1=\left(1-0.065\,\rho/\rho_b\right)h_{\mathrm{ACI}} \quad (14)$$

The statistics properties from Table 4 can be arranged like:

$R_{avg}=0.99996$, Correlation $(r)=0.99891$, Standard Deviation $=0.02383$, Variance $=0.00057$. The corrected total depth can be expressed in another form:

$$h_1=\frac{L}{\beta\, N} \quad (15)$$

where: N is a constant depends on the support condition as shown in table 1 and L is the span of the beam.

Values of (β) are shown in Table 5 for different values of (ρ/ρ_b). Figure 2 shows the effect of the reinforcement indices ratio (ρ/ρ_b) of the correction factor (β), as shown value of (β) increased while the ratio (ρ/ρ_b) is increased. This means that smaller total depth is required with increasing the steel reinforcement area as expected. The best fit equation obtained from Figure 2 to predict (β) in term of reinforcement indices ratio (ρ/ρ_b) is shown below:

$$\beta=1+0.069\,\rho/\rho_b \quad (16)$$

or

$$h_1 = \frac{L}{\left(1+0.069\,\rho/\rho_b\right)N} \tag{17}$$

The results of equations (13) and (16) are used to find the modified depth-span ratios including the effect of the tension reinforcement area for beams and slabs, as shown in tables (6a) and (6b).

Table 4: Correction factor (α).

ρ	ρ/ρ_b	$\alpha = h_1/h$	α_{cal}	$R = \alpha_{cal}/\alpha$
0	0.00	1.0000	1.0000	1.0000
$0.5\rho_b$	0.50	0.9660	0.9675	1.0016
ρ_t	0.63	0.9600	0.9591	0.9991
ρ_{max}	0.72	0.9545	0.9532	0.9986
ρ_b	1.00	0.9345	0.9350	1.0005

Table 5: Correction factor (β).

ρ	ρ/ρ_b	β	β_{cal}	$R = \beta_{cal}/\beta$
0	0.00	1.0000	1.0000	1.0000
$0.5\rho_b$	0.50	1.0352	1.0345	0.9993
ρ_t	0.63	1.0417	1.0435	1.0017
ρ_{max}	0.72	1.0477	1.0497	1.0019
ρ_b	1.00	1.0701	1.0690	0.9990

The statistics properties from Table 5 can be arranged as the following:

$R_{avg} = 1.00039$, Correlation $(r) = 0.99842$, Standard Deviation $= 0.02542$, Variance $= 0.00065$.

Table 6A: Modified minimum thickness (L/N) and N for beams

ρ	ρ/ρ_b	Simply supported	One end continuous (Propped)	Two ends continuous (Fixed ended)	Cantilever
0	0.00	16	18.5	21	8
$0.5\rho_b$	0.50	16.563	19.151	21.739	8.282
ρ_t	0.63	16.667	19.271	21.875	8.333
ρ_{max}	0.72	16.763	19.382	22.001	8.381
ρ_b	1.00	17.121	19.797	22.472	8.560

Table 6B: Modified minimum thickness (L/N) and N for slabs

ρ	ρ/ρ_b	Simply supported	One end continuous (Propped)	Two ends continuous (Fixed ended)	Cantilever
0	0.00	20	24	28	10
$0.5\rho_b$	0.50	20.704	24.845	28.986	10.352
ρ_t	0.63	20.833	25.000	29.167	10.417
ρ_{max}	0.72	20.953	25.144	29.335	10.477
ρ_b	1.00	21.402	25.682	29.963	10.701

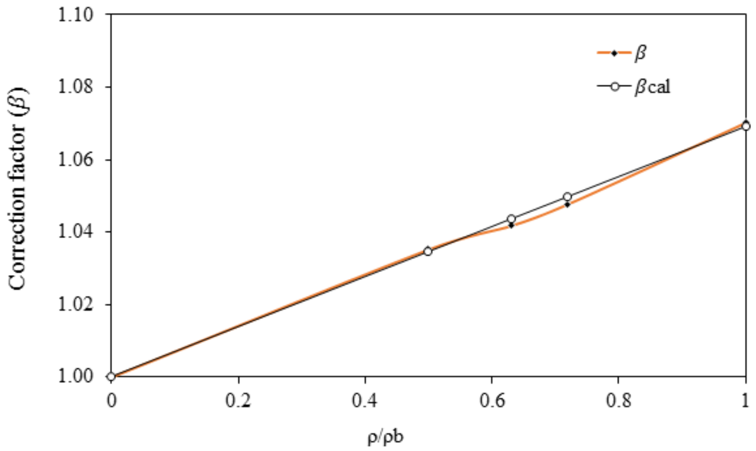

Figure 2: Effect of reinforced indices ratio on the correction factor (β)

4. Conclusion

1. A modification of the ACI code span-depth ratio is suggested in this study to include the effect of tension reinforcement area in addition to the correction for concrete type and yielding strength of steel bars.
2. The correction factors (α and β) are determined in term the reinforcement indices ratio (ρ/ρ_b).
3. The correction factor (α) decreased with increasing the value of the reinforcement indices ratio (ρ/ρ_b).
4. The correction factor (β) increased with increasing the value of the reinforcement indices ratio (ρ/ρ_b).
5. Suitable equations are proposed to predict value of (α) and (β).

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