

Nuclear Gamma-soft Character in ^{128}Ba

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ABSTRACT

We report the properties of gamma soft O(6) of ^{128}Ba isotones with neutron $N = 72$ using Interacting Vector Boson Model (IVBM), interacting Boson Model (IBM-1), Bohr-Mottelson Model (BM), and Doma-El-Gendy (D-G) relation. The first energy level ($E2_1^+$) and ratio $R = E4_1^+ / E2_1^+$ have been investigated which show that ^{128}Ba has gamma-soft character. The curves $E_\gamma / Vs. J$ of E-GOS of even ^{128}Ba nucleus were compared with the standard curves of vibrational, gamma soft and rotational limits. The staggering factors were studied of available measured data of ^{128}Ba nucleus. The yrast levels of this isotope are calculated by the model of VBMI, IBM-1, BM and D-G and they were compared by measured data. The negative parity band of ^{128}Ba was calculated by IVBM and BM model and compared with experimental values.

Keywords: ^{128}Ba , BM, D-G, Ground states band E-GOS, IBM-1, IVBM, Staggering Factor.

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I. INTRODUCTION

The low-lying Beryllium nucleus had been successfully explained nuclear collective characters by Iachello and Arima using IBM-1 [1]. The even-even Beryllium isotope with neutron $N=72$ has been a focus of the nuclear structure of many theoretical and experimental investigations. In the first beginning the collective states can be describes by a system of identical bosons N_B . These are S-boson ($L=0$) and d-boson ($L=2$). There is no discrepancy between neutron and proton in IBM-1. There are three dynamical symmetries indicated by gamma-soft O(6), vibration U(5) and rotational SU(3) [2]. The rotational energy E as a function of $J(J+1)$ of a nucleus was found by Bohr and Mottelson [3]. The ground and octupole bands of nuclei were explained by *Ganev et al* using the interacting vector boson model (IVBM) [4]. Doma and EL-Gendy (D-G) developed a modern relation between moment of inertia and the spin of the nucleus [5].

The ^{128}Ba nucleus has neutron numbers 72 and atomic numbers 56 respectively. They belongs near to closed shell ^{132}Sn (magic number $Z=50$ and $N=82$). The external forms of even-even nucleus of Ba, with neutron number 72 have

$\pi g_{9/2}^{+,6}$ (6 proton particles) and $\nu h_{11/2}^{-10}$ (10 neutron holes) close to double magic nucleons ^{132}Sn . This configuration has been investigated the ground state structure from spherical to deformed symmetry. The evolution of yrast levels were studied around $A=110$ region by Regan et al. [6].

Recently, the properties of the yrast level were studied in Pd isotopes with even neutron $N=54-64$ [7]. The electromagnetic reduced transition strength of Cd isotopes with $N=66-74$ were investigated [8]. The $B(E2)$ values of yrast band of even $^{102-112}\text{Pd}$ and $^{96-102}\text{Ru}$ isotopes were investigated by IBM-1[9]-[11]. The lower level of ^{184}W and ^{184}Os nuclei were investigated [12].

The nobility of present work particularly focuses the structure of the yrast band ($E2^+$) which equals to 0.1, 0.3 and 0.5 MeV and the ratios $R = E4_1^+ / E2_1^+$ which obeys $2.0 < R < 2.4$ for vibrator, $2.4 < R < 3.0$ for gamma soft and $3.0 < R < 3.3$ for rotator, the E-GOS curve, staggering factor, the negative parity band of even-even ^{128}Ba nucleus by IVBM, IBM-1, BM, and D-G methods.

II. MATERIALS AND METHOD

A. Interacting Boson Model

The IBM gives occupation to truncated model space for nuclei with N number of nucleons. It provides a quantitative description of indistinguishable particles with forming pairs of $L = 0$ and 2.

The Hamiltonian of IBM-1 is given [13]:

$$H = \sum_{i=1}^N \varepsilon_i + \sum_{i < j}^N V_{ij} \quad (1)$$

Here, ε indicates energy of boson, V_{ij} indicates potential energy of boson between i and j .

Hamiltonian is given from multi-pole form [14]:

$$H = \varepsilon \hat{n}_d + a_0 (\hat{P} \cdot \hat{P}) + a_1 (\hat{L} \cdot \hat{L}) + a_2 (\hat{Q} \cdot \hat{Q}) + a_3 (\hat{T}_3 \cdot \hat{T}_3) + a_4 (\hat{T}_4 \cdot \hat{T}_4) \quad (2)$$

Here P (the pairing operator), Q (quadrupole operator), \hat{n}_d (number of d boson), L (operator of angular momentum), and T_3 (octupole operators) and T_4 (hexadecapole operators).

The Hamiltonian starting with $U(6)$ and complete with group $O(2)$ as given in Eq.(2) is bringing to a lower state of three limits, gamma-soft $O(6)$, vibration $U(5)$ and rotational $SU(3)$ nuclei [15]. It is noted that the limit in the $O(6)$, $SU(3)$ and $U(5)$ the parameters are a_0 , a_2 , and ε , respectively.

For the three limits, Hamiltonian and eigen-values [16]: $U(5)$:

$$\hat{H}_{U(5)} = \varepsilon \hat{n}_d + a_1 \hat{L} \cdot \hat{L} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4 \quad (3)$$

$$E(n_d, \nu, L) = \varepsilon n_d + K_1 n_d (n_d + 4) + K_4 \nu(\nu + 3) + K_5 L(L + 1)$$

with

$$K_1 = 1/12 a_1$$

$$K_4 = -1/10 a_1 + 1/7 a_3 - 3/70 a_4$$

$$K_5 = -1/14 a_3 + 1/14 a_4$$

$O(6)$:

$$\hat{H}_{O(6)} = a_0 \hat{P} \cdot \hat{P} + a_1 \hat{L} \cdot \hat{L} + a_3 \hat{T}_3 \cdot \hat{T}_3 \quad (4)$$

$$E(\sigma, \nu, L) = K_3 [(N(N + 4) - \sigma(\sigma + 4))] + K_4 \nu(\nu + 3) + K_5 L(L + 1)$$

with

$$K_3 = 1/4 a_0$$

$$K_4 = 1/2 a_3$$

$$K_5 = -1/10 a_3 + a_1$$

$SU(3)$:

$$\hat{H}_{SU(3)} = a_1 \hat{L} \cdot \hat{L} + a_2 (\hat{Q} \cdot \hat{Q}) \quad (5)$$

$$E(\lambda, \mu, L) = K_2 (\lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu)) + K_5 L(L + 1)$$

with

$$K_2 = 1/2 a_2$$

$$K_5 = a_1 - 3/8 a_2$$

K_1 , K_2 , K_3 , K_4 , and K_5 are other strength parameters.

Then applying particular limit of symmetry ($O(6)$, $SU(3)$, and $U(5)$) to determine the frame of a set of nuclei is more advantageous than full Hamiltonian of IBM-1. It comprises multi-free parameters to fit the structure of nuclei.

For three limits, $R = E_\gamma / J$ and J are given [17], [18]:

$$\text{Vibrator: } R = \frac{\hbar\omega}{J} \rightarrow 0 \text{ when } J \rightarrow \infty \quad (6)$$

$$\gamma \text{-soft: } R = \frac{E_2^+}{4} \left(1 + \frac{2}{J}\right) \rightarrow \frac{E_2^+}{4} \text{ when } J \rightarrow \infty \quad (7)$$

$$\text{Rotor: } R = \frac{\hbar^2}{2\mathcal{I}} \left(4 - \frac{2}{J}\right) \rightarrow 4 \frac{\hbar^2}{2\mathcal{I}} \text{ when } J \rightarrow \infty \quad (8)$$

where \mathcal{I} is the moment of inertia.

For the yrast states of two or three limits, their eigenvalues are given by [19]:

$U(5)$ - $O(6)$:

$$E(\varepsilon, n_d, \tau, L) = \varepsilon \hat{n}_d + K_1 n_d (n_d + 4) + K_4 \tau(\tau + 3) + K_5 L(L + 1) \quad (9)$$

$U(5)$ - $SU(3)$:

$$E(\varepsilon, \lambda, L) = \varepsilon \hat{n}_d + K_2 (\lambda^2 + 3(\lambda + \mu)) + K_5 L(L + 1) \quad (10)$$

$O(6)$ - $SU(3)$:

$$E(\tau, \lambda, L) = \varepsilon \hat{n}_d + K_2 (\lambda^2 + 3(\lambda + \mu)) + K_4 \tau(\tau + 3) + K_5 L(L + 1) \quad (11)$$

The equation of odd-even staggering [20]:

$$\Delta E_{1,\gamma}(J) = \frac{1}{16} (6E_{1,\gamma}(J) - 4E_{1,\gamma}(J-1) - 4E_{1,\gamma}(J+1) + E_{1,\gamma}(J-2) + E_{1,\gamma}(J+2)) \quad (12)$$

where the transition energy is:

$$E_{1,\gamma}(J) = E(J+1) - E(J) \quad (13)$$

The symmetry for the excited states of even-even nuclei [21], [22]:

$$r\left(\frac{J+2}{J}\right) = \left[\left(\frac{R(J+2)}{J} \right)_{\text{exp.}} \frac{(J+2)}{J} \right] \times \frac{J(J+1)}{2(J+2)} \quad (14)$$

The value of (r) has changed from 0.1 to 0.35, 0.4 to 0.6 and 0.6 to 1.0 for vibrational, transitional, and rotational of yrast bands of even-even nuclei.

In BM model, small J, the energy E can be expanded in power of J(J+1) The GSB and the NPB levels are given by [3]-[23]:

$$E(J) = AJ(J+1) - BJ^2(J+1)^2 + CJ^3(J+1)^3 \quad (15)$$

$$E(J) = E_0 + AJ(J+1) - BJ^2(J+1)^2 + CJ^3(J+1)^3 \quad (16)$$

E_0 is the band head energy of the NPB. The A, B, and C coefficients.

The equation of eigenvalues for the GSB in IVBM [4]:

$$E(J) = \beta J(J+1) + \gamma J \quad (17)$$

The equation of eigenvalues for the NPB in IVBM [4]:

$$E(J) = \beta J(J+1) + (\gamma + \eta)J + \zeta \quad (18)$$

β , γ and η , ζ evaluated from a fit to the positive GSB and NPB respectively. The equations (17), (18) shows that the Eigen states of the GSB and NPB consist of rotational $J(J+1)$ and vibration J modes. Doma and El-Gendy derived a new equation for rotational energy, that depends on J and g [24].

$$E(J) = AJ(J+1) \quad (19)$$

$$[1 + DJ(J+1)/(1 - CJ(J+1))]^{-1}$$

$$A = \frac{\hbar^2}{2g}$$

The D and C values are found from a fit to the ground states band.

III. RESULTS AND DISCUSSION

The excitation energy states give primary information regarding gamma-soft symmetry of a nucleus. In the present study, the first excited states ($E2_1^+$) and the ratios of $R = E4_1^+/E2_1^+$ of ^{128}Ba isotone were investigated and the data were listed in table 1, which shows that ^{128}Ba has the gamma-soft property.

TABLE I: THE MEASURED VALUES [25], OF $E2_1^+$ IN (KEV) AND THE

RATIOS $R = (E4_1^+ / E2_1^+)$ OF ^{128}Ba EVEN-EVEN NUCLEUS

| Nucleus | ^{128}Ba |
|-------------------------|-------------------|
| $E2_1^+$ | 284.0 |
| $R = (E4_1^+ / E2_1^+)$ | 2.69 |

Table I does not give complete information about the property of the nucleus at their different excited states, which

may change, which is why the E-GOS of the measured gamma energy [25] is drawn in Fig. 1. To estimate the change in each nucleus property, which shows that ^{128}Ba isotone change from the fast to the slow decreasing and then increasing slowly which indicates that this isotone has changeable properties along the yrast band. Study on Nuclear Structure of even-even ^{128}Ba Nuclei for $N = 72$ was presented to first International Conference on Advances in Material Science [26].

TABLE II: THE RATIO $R(J+2/J)$ Vs. $J(\hbar)$

| Nucleus | J = 2 | J = 4 | J = 6 | J = 8 | J = 10 |
|-------------------|-------|-------|-------|-------|--------|
| ^{128}Ba | 0.515 | 0.573 | 0.584 | 0.569 | 0.43 |

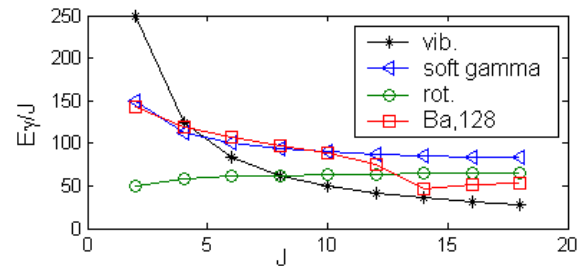


Fig. 1. The E-GOS curves E_γ / J (keV / h) Vs. J (h) of even ^{128}Ba nucleus compared with the standard curves of vibrator, gamma soft and rotator limits.

To confirm the properties of this nucleus, the staggering modes of the measured energy levels [25] of ^{128}Ba , has shown in Fig. 2. The staggering of ^{128}Ba approach the vanishing values and increasing again indicating a change in their properties. We do not draw the staggering modes of ^{128}Ba NPB, because there are no measured values of so them.

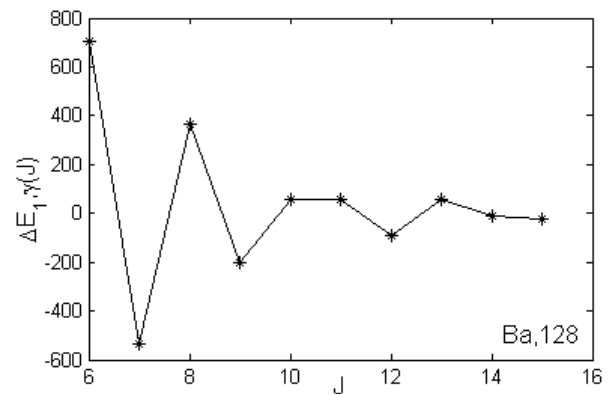
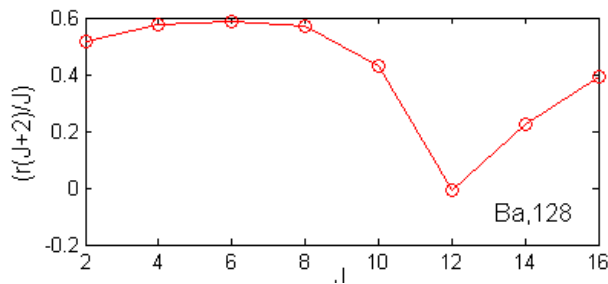


Fig. 2: The staggering factor $\Delta E_{1,\gamma}(J)$ (keV) Vs. J (h) of ^{128}Ba nucleus.

The first excited state, the ratio R, E-GOS curve and the staggering curves of even ^{128}Ba isotone do not give the type of the change in their properties along the yrast band. Therefore, the relation between the measured G.S.B. [25], [26] of each nucleus was investigated. This discussion is for the G.S.B only, since we calculate this band. Table 2 and Fig. 3 show the $r((J+2)/J)$ values $-0.069 \leq r \leq 0.608$, are ^{128}Ba (the gamma-soft property).

Table II, the ratio $r(J+2/J)$ Vs. $J(\hbar)$ of the available ground states band of even ^{128}Ba isotone.

Fig. 3. The ratio $r((J+2)/J)$ Vs. J (\hbar) of the yrast band of ^{128}Ba nucleus.

The energy levels of the ground states and negative parity bands of ^{128}Ba isotone were calculated using BM, IBM-1, IVBM and D-G methods by MATLAB 6.5 program codes. The bosons number and the values of the best parameters to calculate yrast levels of those nuclei are listed in Table III. The values of $\hbar^2/2\mathcal{I}$ are 47.33, of ^{128}Ba , respectively do agree with the values of parameters of these isotones, because the low-lying states of these nuclei have a rotational property.

TABLE III: IBM-1, IVBM, BM AND D-G PARAMETERS IN (keV) USED TO CALCULATE THE YRAST LEVELS OF ^{128}Ba EVEN-EVEN ISOTONE

| | | IBM1 | | | | | IVBM | |
|-------------------|-------|---------------|-------|-------|-------|--------|---------|----------|
| Nucl. | N_b | \mathcal{E} | K_1 | K_2 | K_4 | K_5 | β | γ |
| ^{128}Ba | 8 | 140 | 75.89 | 0.16 | 98.0 | 12.83 | 21.93 | 76 |
| | | BM | | | DG | | | |
| Nucl. | | A | B | C | A | C | D | |
| ^{128}Ba | | 42.51 | -0.24 | 0.001 | 41.6 | -0.001 | 0.005 | |

TABLE IV: IVBM AND BM PARAMETERS IN (keV) USED TO CALCULATE THE NEGATIVE PARITY BAND ENERGY LEVELS OF EVEN-EVEN ^{128}Ba ISOTONE

| | | IVBM | | | BM | |
|-------------------|--------|--------|-------|--------|-------------|-------------|
| Nucl. | η | ξ | E_0 | A | $B*10^{-3}$ | $C*10^{-5}$ |
| ^{128}Ba | -248.6 | 2374.6 | 1520 | 0.0163 | -0.0125 | 0.00252 |

The measured [25] and calculated energies of the G.S.B of IBM-1, IVBM, BM and DG are listed in table V and the negative parity states of IVBM and BM are listed in table VI. It is clearly observed that the calculated values are well with the measured values of ^{128}Ba nucleus.

TABLE V: THE MEASURED AND CALCULATED ENERGY LEVELS (keV) OF THE YRAST BAND IN ^{128}Ba NUCLEUS

| J_1^+ | Exp. | Calculated | | | |
|---------|--------|------------|--------|--------|--------|
| | | IBM-1 | IVBM | BM | DG |
| 2 | 284.0 | 284 | 284 | 246.8 | 241.9 |
| 4 | 763.2 | 763.2 | 743.4 | 763.0 | 754.8 |
| 6 | 1400.9 | 1389.3 | 1378.3 | 1437.8 | 1446.8 |
| 8 | 2188.5 | 2162.4 | 2188.5 | 2188.5 | 2231.2 |
| 10 | 3082.3 | 3082.3 | 3174.2 | 3082.3 | 3050.8 |

TABLE VI: THE MEASURED AND CALCULATED ENERGY LEVELS (keV) OF NEGATIVE PARITY BAND OF EVEN-EVEN ^{128}Ba NUCLEUS

| | | ^{128}Ba | |
|-----|------|-------------------|--------|
| J | Exp. | Cal. | |
| | | IVBM | BM |
| 5 | 2039 | 2170.6 | 1997.4 |
| 7 | 2369 | 2369 | 2369 |
| 9 | 2906 | 2796.8 | 2900.9 |
| 11 | 3507 | 3373.2 | 3507 |
| 13 | 4218 | 4124.9 | 4218 |
| 15 | 5052 | 5052 | 5052 |

IV. CONCLUSIONS

In conclusion, the measured values of ^{128}Ba nucleus of the first excited states $E2_1^+$ is 284.0 keV and the ratios of the second to the first excited states $E4_1^+ / E2_1^+$ is 2.69 which give the property of the information of gamma soft character. The E-GOS and the staggering factor $\Delta E_{1,\gamma}(J)$ give good information about the change of the property that may occur along their states. The staggering of ^{128}Ba nucleus approach the vanishing values and increasing again indicating a change in their properties. The relation between the energy ratio of the $J+2$ state to J state $r\left(\frac{(J+2)}{J}\right)$ as a function of J state gives properly the exact property of the nucleus. This study insured that ^{128}Ba nucleus has a vibration- γ -soft U (5)-O (6) property. The IBM-1, IVBM, BM and D-G have been applied to calculate the energy levels of the ground states band, the IVBM and BM have been applied to calculate the energy levels of the negative parity band of nuclei under consideration, the calculated energy levels are in good agreement with the measured values with different accuracy duo to the different suit of each model to each nucleus.

DECLARATION OF STATEMENT

All authors are equally contributed to this work.

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